compound interest – interest that builds off previous values of final amounts

Example 1 **Comparing Simple and Compound Interest**

Suppose that \$5,000 is invested for 3 years at 8%.

(a) Find the amount of simple interest.

(b) Find the compound interest if interest is calculated once per year.

$I = ?$
$P = \$5000$
$r = .08$
$t = 3\text{ years}$

a.) $I = Prt$

$I = (5000)(.08)(3)$

$= \$1200$

b.) Year 1 $\Rightarrow$ $I = Prt$

$= (5000)(.08)(1)$

$= \$400$

$t = 1$ | $A = P + I$

$= 5000 + 400$

$= \$5,400$

Year 2 $\Rightarrow$ $I = Prt$

$= (5,400)(.08)(1)$

$= \$432$

$A = P + I$

$= 5400 + 432$

$= \$5832$

Year 3 $\Rightarrow$ $I = Prt$

$= (5832)(.08)(1)$

$= \$466.56$

$A = P + I$

$= 5832 + 466.56$

$= \$6298.56$

Total Interest (Compound)

$I = \$400 + \$432 + 466.56$

$= \$1298.56$

Developing Compound Interest Formula

$t = 1$

$A = P + I = P(1 + rt)$

Year 1 $\Rightarrow 5400 = 5000(1 + (.08)) = 5000(1.08)$

Year 2 $\Rightarrow 5832 = 5400(1.08) = 5000(1.08)(1.08) =$

$= 5000(1.08)^2$

Year 3 $\Rightarrow 6298.56 = 5832(1.08) = 5000(1.08)^2(1.08)$

$= 5000(1.08)^3$

Year 223 $\Rightarrow 5000(1.08)^{223}$

$A = P \cdot (1 + r)^t$

annual compound interest

exponential growth.

Formula for Computing Compound Interest

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

where A is the future value (principal + interest)

r is the yearly interest rate in decimal form

n is the number of times per year the interest is compounded

t is the term of the investment in years

Example 3 **Computing Compound Interest**

Find the interest on $7,000 compounded quarterly at 3% for 5 years.

4x per year $\rightarrow n=4$

$A = ?$

$P = \$7000$

$r = 0.03$

$t = 5$ years

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A = (7000)\left(1+\frac{(0.03)}{(4)}\right)^{(4)(5)}$$

$$A = (7000)\left(\frac{4}{4}+\frac{0.03}{4}\right)^{20}$$

$$A = (7000)\left(\frac{4.03}{4}\right)^{20}$$

$$A = (7000)(1.0075)^{20}$$

$$A = (7,000)(1.16118)$$

$$A = \$8128.26$$

Still Need to find the interest

$$A = P + I$$

$$I = A - P$$

$$I = 8128.26 - 7000$$

$$I = \$1128.26$$

Example 4 **Computing Compound Interest**

Find the interest on \$11,000 compounded daily at 5% for 6 years. Assume a 365-day year.

$P = \$11{,}000$ $t = 6$ years

$r = 0.05$ $n = 365$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$= (11{,}000)\left(1+\frac{(0.05)}{(365)}\right)^{(365)(6)}$$

$$= \$14{,}848.14$$ ← final value

* Not Interest

$$A = P + I$$

$$(\$14848.14) = (11{,}000) + I$$

$$-11{,}000.00 \quad -11{,}000$$

$$\$3{,}848.14 = I$$ ← interest

Example 5 **Finding the Time Needed to Reach an Investment Goal**

If you want to save $5,000 before buying your first new car, and you have $3,000 right now to invest at 3% interest compounded monthly, how long will you have to wait?

$A = 5000$

$P = 3000$

$r = 0.03$

$n = 12$

$t = ?$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$\frac{(5000)}{3000} = \frac{\cancel{(3000)} \cdot \left(1 + \frac{(0.03)}{(12)}\right)^{12t}}{\cancel{3000}}$$

$$\frac{5}{3} = \left(1 + \frac{0.03}{12}\right)^{12t}$$

$$\frac{5}{3} = \left(\frac{12.03}{12}\right)^{12t}$$

$$\log\left(\frac{5}{3}\right) = \log\left(\left(\frac{12.03}{12}\right)^{12t}\right)$$

$$\frac{\log\left(\frac{5}{3}\right)}{12\log\left(\frac{12.03}{12}\right)} = \frac{\cancel{12}t \cdot \cancel{\log\left(\frac{12.03}{12}\right)}}{\cancel{12}\cancel{\log\left(\frac{12.03}{12}\right)}}$$

$$t = \frac{\log\left(\frac{5}{3}\right)}{12\log\left(\frac{12.03}{12}\right)} \approx 17 \text{ years } 1 \text{ month}$$

The **effective rate** (also known as the **annual yield**) is the simple interest rate which would yield the same future value over 1 year as the compound interest rate.

The next formula can be used to calculate the effective interest rate.

Formula for Effective Interest Rate

$$E = \left(1 + \frac{r}{n}\right)^n - 1$$

where

E = effective rate

n = number of periods per year the interest is calculated

r = interest rate per year (i.e., stated rate)

Example 6 Finding Effective Interest Rate

Find the effective interest rate when the stated rate is 4% and the interest is compounded weekly, then describe what your result means.

$r = .04$

$n = 52$

$$E = \left(1 + \frac{r}{n}\right)^n - 1$$

$$E = \left(1 + \frac{(.04)}{(52)}\right)^{52} - 1$$

$$= \left(\frac{52}{52} + \frac{.04}{52}\right)^{52} - 1$$

$$= \left(\frac{52.04}{52}\right)^{52} - 1 = .0407\ldots$$

Effective Rate 4.1%

Example 7 Comparing the Effective Rate of Two Investments

$E = \left(1+\frac{r}{n}\right)^n - 1$

Which savings account is a better investment: 6.2% compounded daily or 6.25% compounded semiannually?

6.2% Daily $n = 365$, $r = .062$

$$E = \left(1 + \frac{r}{n}\right)^n - 1$$

$$= \left(1 + \frac{.062}{365}\right)^{365} - 1$$

$$\approx .0644\ldots \rightarrow 6.4\%$$

6.25% semiannually $r = .062$ $n = 2$

$$E = \left(1 + \frac{r}{n}\right)^n - 1$$

$$E = \left(1 + \frac{.0625}{2}\right)^2 - 1$$

$$E \approx 0.0634\ldots \rightarrow 6.3\%$$

* desmos.com/c

* savings → higher interest rate is better

∴ Best option is 6.2% daily.