Sample Spaces

Processes such as flipping a coin, rolling a die, or drawing a card from a deck are called *probability experiments.*

Example 1 **Finding Probability from Percent Chance**

When you flip a coin, what's the percent chance that it will land heads up? What's the probability?

% chance – 50%

Probability – $\frac{1}{2}$

> A **probability experiment** is a process that leads to well-defined results called outcomes. An **outcome** is the result of a single trial of a probability experiment.

Some examples of a trial are flipping a coin once, rolling a single die, and drawing one card from a deck. When a coin is tossed, there are two possible outcomes: heads or tails. When rolling a single die, there are six possible outcomes: 1, 2, 3, 4, 5, or 6.

In a probability experiment, we can predict what outcomes are possible, but we can't predict with certainty which one will occur. We say that the outcomes occur at **random.** In any experiment, the set of all possible outcomes is called the *sample space.*

> A **sample space** is the set of all possible outcomes of a probability experiment.

different outcomes

coin $S = \{H, T\}$

sample space

6-sided die $S = \{1, 2, 3, 4, 5, 6\} = \{1, 2, \ldots, 5, 6\}$

An **event** is any subset of the sample space of a probability experiment.

6-sided die $S = \{1, 2, 3, 4, 5, 6\}$

e.g. event

roll a 6 - also an outcome

roll an even number: $E = \{2, 4, 6\}$

roll anything less than 7: $E = \{1, 2, 3, 4, 5, 6\}$

roll a 0: $\{\ \}$ or $\emptyset$

Theoretical Probability

Now we're ready to specifically define what is meant by probability. The first type we'll study is called **theoretical probability.** The goal is to determine all of the possible outcomes in a sample space and determine the probability, or likelihood, of an event occurring without actually performing experiments. There is one key assumption we make in theoretical probability: that every outcome in a sample space is equally likely. For example, when a single die is rolled, we assume that each number is equally likely to come up. When a card is chosen from a deck of 52 cards, we assume that each card has the same probability of being drawn.

Math Note

Theoretical probability is also called classical probability because it was the first type studied in the 17th and 18th centuries.

Formula for Theoretical Probability

Let E be an event in the sample space S, n(E) be the number of outcomes in E, and n(S) the number of outcomes in S. The probability of E is

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{Number of outcomes in } E}{\text{Number of outcomes in } S}$$

Example 2 Computing Theoretical Probabilities

A single die is rolled. For the three events listed below, without calculating probabilities, put them in order from least to most likely based on an educated guess. Then compute each probability.

(a) A 5

(b) A number less than 5

(c) An odd number

$S = \{1, 2, 3, 4, 5, 6\}$

$n(S) = |S| = 6$

less than 5 , odd number, a 5

$E = \{1, 2, 3, 4\}$ $\quad E = \{1, 3, 5\}$ $\quad E = \{5\}$

$n(E) = 4$ $\quad n(E) = 3$ $\quad n(E) = 1$

$P(\#\text{less than } 5) = \frac{4}{6}$ $\quad P(\text{odd}) = \frac{3}{6}$ $\quad P(5) = \frac{1}{6}$

Example 3 Computing Theoretical Probabilities

Two coins are flipped. Find the probability of getting

(a) Two heads.

(b) At least one head.

(c) At most one head.

coin 1 $\quad$ coin 2

$S = \{HH, TH, HT, TT\}$

$|S| = 4$

$P(2H) = \frac{1}{4}$

$E(H \geq 1) = \{HH, HT, TH\}$ $\quad P(H \geq 1) = \frac{3}{4}$

$|E| = 3$

$E(H \le 1) = \{HT, TH, TT\}$ $P(H \le 1) = \frac{3}{4}$

$|E| = 3$

1. **Probability is never negative.** Both $n(E)$ and $n(S)$ have to be zero or positive, so we can't get a negative number by dividing them.
2. **Probability is never greater than one.** An event is a subset of the sample space, so there can't be more outcomes in any event than in the entire sample space; that means the numerator is less than or equal to the denominator in the probability formula.
3. **When an event can't possibly occur, its probability is zero. When an event is certain to occur, the probability is one.** If an event can't occur, then none of the outcomes in the sample space satisfy it and $n(E) = 0$. If an event has to occur, then every outcome in the sample space satisfies it and $n(E) = n(S)$, so $n(E)/n(S) = 1$.
4. **If you add the probabilities for every outcome in the sample space, the result is always one.** For example, when a die is rolled, each of the six outcomes has probability $\frac{1}{6}$ so the sum of the probabilities for those six outcomes is one.

Example 4	Finding a Probability Using Complements

Of the next 32 trials on the docket in a county court, 5 are homicides, 12 are drug offenses, 6 are assaults, and 9 are property crimes. If jurors are assigned to trials randomly, what's the probability that a given juror won't get a homicide case? Find the probability using both the formula for theoretical probability on page 558 and the formula for complements. Then compare the two methods.

$$P(\text{not homicide}) = P(DO) + P(A) + P(PC)$$

$$= \frac{12}{32} + \frac{6}{32} + \frac{9}{32}$$

$$= \frac{27}{32}$$

$$P(\text{not homicide}) = \frac{12+6+9}{32} = \frac{27}{32}$$

$$P(\text{not homicide}) = 1 - P(\text{homicide})$$

$$= 1 - \frac{5}{32}$$

$$= \frac{32}{32} - \frac{5}{32} = \frac{27}{32}$$

$*\ P(A) + P(\text{not } A) = 1$

Empirical Probability

Lecture: Useful Facts about Probability

The second approach to probability we will study is to compute it using experimental data, rather than counting equally likely outcomes. For example, suppose 100 games into the season, your favorite baseball team has won 60 games and lost 40. You might reasonably guess that since they've won 60 of their 100 games so far, the probability of them winning any given game is about 60/100, or 0.6. This type of probability is called **empirical probability,** and is based on *observed frequencies*—that is, the number of times a particular event has occurred out of a certain number of trials. In this case, the observed frequency of wins is 60, the observed frequency of losses is 40, and the total number of trials is 60 + 40 = 100.

Formula for Empirical Probability

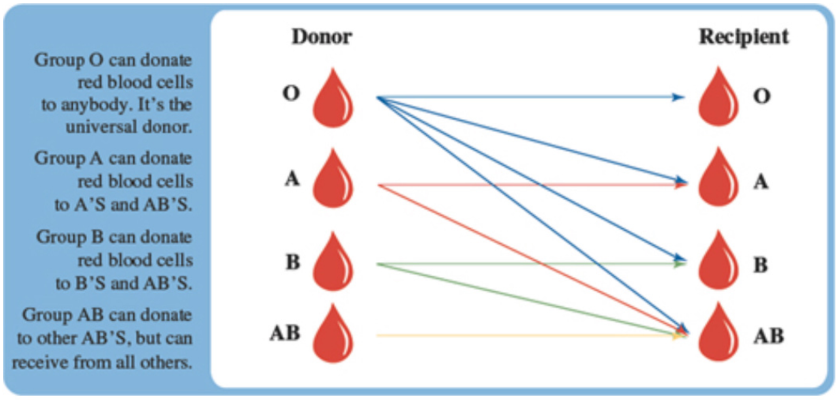
$$P(E) = \frac{\text{Observed frequency of the specific event } (f)}{\text{Total number of trials } (n)} = \frac{f}{n}$$

Math Note

The total number of trials is the sum of all observed frequencies

Example 5 **Computing Empirical Probabilities**

Everyone has a blood type that falls into one of four categories: O, A, B, or AB. When a blood transfusion is needed in an emergency medical situation, you might think that you'd have to get the same type as your blood, but that's not actually the case. The chart below describes which types can donate, and receive, various blood types.



In a random sample of 500 people, 210 had type O blood, 223 had type A, 51 had type B, and 16 had type AB. First, set up a frequency distribution. Then use it to answer the following questions.

(a) What's the probability that a randomly selected patient can receive only one type of blood?

(b) If an accident victim has type B blood, and an EMT on site is the only person willing to donate blood, what's the probability that it will be compatible with the victim?

(c) If a person with blood type A gets a transfusion, what's the probability that the donor had type O?

(d) Find the probability that a randomly selected donor's blood can be given to patients of more than just one blood type.

Blood Type	Frequency
O	210
A	223
B	51
AB	16
	500

Total

a) $P(O) = \frac{210}{500} = \frac{21}{50}$

b.) $P(O) + P(B)$

$\frac{210}{500} + \frac{51}{500} = \frac{261}{500}$

c.) $P(O \mid \text{can donate to } A) = \frac{210}{210+223} = \frac{210}{433}$

d.) $P(\text{can give to more than one blood type}) =$

$= \frac{210+223+51}{500} = \frac{484}{500}$

$= 1 - P(AB) = 1 - \left(\frac{16}{500}\right) = \frac{500-16}{500} = \frac{484}{500}$