MAT 2680 Section D669 **Name (Print):** ____________________

Fall 2015

Professor K. Poirier

Final Exam

December 21 10:00-11:15am **Time Limit: 75 Minutes**

This test contains 6 pages. Check to see if any pages are missing. Print your name on the top of this page, and put your initials on the top of every page, in case the pages become separated.

Answer 5 of the following 6 questions. (If all 6 are answered, only the first 5 will be graded.)

You may use a calculator on this test. No other aids are allowed. Show all your work for full credit.

Total: 50 points including one point per question for style (this includes showing all work, using all notation correctly, and presenting the work logically).

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1. (10 points) An object weighing 256 lb is dropped from rest in a medium that exerts a resistive force with magnitude proportional to the square of the speed. The magnitude of the resisting force is 1 lb when $|v| = 4$ ft/s. Find v for $t > 0$, and find its terminal velocity.

2. (10 points) A 96 lb weight is attached to a spring with constant 12 lb/ft. Find the steady state component of the displacement if the mass is subjected to an external force of $F(t) = 18\cos(t) - 9\sin(t)$ lb an a dashpot supplies 12 lb of damping for each ft/sec of velocity.

3. (10 points) Use the method of undetermined coefficients to find the general solution:

$$y'' - 2y' + y = e^{-x}(2 + 3x)$$

4. (10 points) Consider the following initial value problem:

$$y'' + 3y' + 2y = e^t, \quad y(0) = 1,\ y'(0) = -6$$

(a) Use the Laplace transform to solve the initial value problem.

(b) Verify that your solution in part (a) is correct.

5. (10 points) Consider the following differential equation:

$$y'' - 2y' + y = 14x^{\frac{3}{2}}e^{x}$$

(a) Find a fundamental set of solutions of the complementary homogeneous equation.

(b) Use variation of parameters with this fundamental set of solutions to find a particular solution of the above equation.

6. (10 points) Find the first five coefficients a_0, a_1, a_2, a_3, a_4 of the series solution, centered at $x = 0$, for the initial value problem.

$$(1 + x^2)y'' + xy' + y = 0, \quad y(0) = 2,\ y'(0) = -2$$