Roller Coaster

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Abstract:

Riding roller coasters is an exciting and thrilling way to spend your day. You may have many questions about them, for instance how is a roller coaster built? We will answer this question using calculus. The question has been adapted from Stewart, *Single Variable Calculus: Early Transcendentals* .

Introduction:

A roller coaster is a "small gravity railroad having a train with open cars that move along a high, sharply winding trestle built with steep inclines that produce sudden, speedy plunges for thrill seeking passengers." Building a roller coaster has many components to it:

- Slope
- Turns/curves
- Speed
- Size

In this project we will study some conditions for building a roller coaster for it to function.

Research Question:

The first part of a roller coaster involves an ascent and descent. Given an ascent of a certain steepness and a descent of a certain steepness, what path can the roller coaster take in between so that it:

1)Looks smooth

2) And also feels smooth

Methodology:

a) This research models the structure of a real roller coaster. For it to look smooth, we find a quadratic function in between the linear functions modeling the ascent and descent given that:

- Slope of the ascent=0.8
- Slope of the drop=-1.6
- Distance between transition points is 100 ft

b)To make the roller coaster feel smooth, it is necessary to improve the first condition because the function modeling the roller coaster will not have a continuous second derivative. For that, we found functions whose first two derivatives agree and connected them between the linear and quadratic functions

Result:

a)Finding f(x), L1(x), and L2(x):

At x=0, $f'(0)=2a(0)+b=0.8$

$b=0.8$

At x=100 $f'(100)=200a+0.8=-1.6$

$a=-0.012$

And $f(0)=+b(0)+c$ $c=0$

$f(x)=-$

With derivatives: $f'(x)=-0.024x+0.8$

$f''(x)=-0.024$

We found also

L1(x)=0.8x

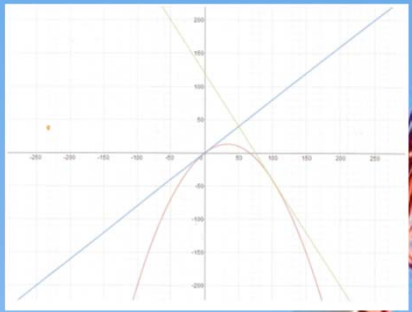
L2(x)=-1.6x+120

The following are displayed on the graph:

$f(x)=-0.012x^2+0.8x$

L1(x)=0.8x

L2(x)=-1.6x+120



b) In part a ,the roller coaster would not feel smooth because the second derivative is not continuous at transition points. In order for the roller coaster to feel smooth, we insert functions g(x) and h(x) between the linear and quadratic functions.

So $L_1(x)=0.8x$ for $x<0$

$g(x)=kx^3+lx^2+mx+n$ $0\le x<10$

$q(x)=ax^2+bx+c$ $10\le x\le 90$

$h(x)=px^3+qx^2+rx+s$ $90<x\le 100$

$L_2(x)=-1.6x+120$ $x>100$

Derivatives:

- L11'=0.8 L11"=0
- g'(x)= 3k+2lx+m g"(x)=6kx+2l
- q'(x)=2ax+b q"(x)=2a
- h'(x)=3p+2qx+r h"(x)=6px+2q
- L2'(x)=-1.6 L2"=0

We found at transition points:

At x=0

L11(0)=g(0) 0.8(0)= k(0)+l(0)+m(0)+n n=0

L11'(0)=g'(0) 0.8=3k+2l(0)+m m=0.8

L11"(0)=g"(0) 0=6k(0)+2l l=0

At x=10

g (10)=q(10) 1000k+100l+10m+n=100a+10b+c

g'(10)=q'(10) 300k+20l+m= 20a+b

g" (10)=q"(10) 60k+2l=2a

At x=90

q (90)=h(90) 8100a+90b+c=729000p+8100q+90r+s

q'(90)=h'(90) 180a+b=24300p+180q+r

q"(90)=h"(90) 2a=540p+2q

At x=100

h(100)=L2(100) 1000000p+10000q+100r+s=-40

h'(100)=L2'(100) 30000p+200q+r=-1.6

h"(100)=L22"(100) 600p+2q=0

We solve these 12 equations that have 11 unknowns with computer algebra system

a= $-\frac{1}{75}$ b= $\frac{14}{15}$ c= $-\frac{4}{9}$

K= $-\frac{1}{2250}$ l=0 m= $\frac{8}{10}$ n=0

p= $\frac{1}{2250}$ q= $-\frac{2}{15}$ r= $\frac{176}{15}$ s= $-\frac{2920}{9}$

The following are the formulas that make the roller coaster feel smooth, and are also displayed on the graph below:

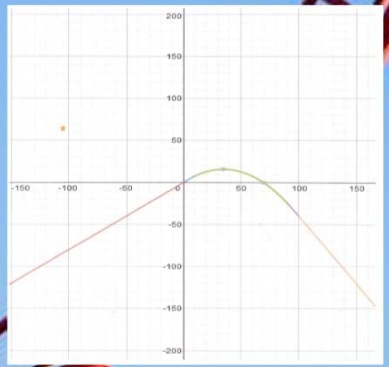
L1(x)=0.8x

$g(x)=(-\frac{1}{2250}^{3})+\frac{8}{10}x$

$q(x)=(-\frac{1}{75}^{2})+\frac{14}{15}x^{\frac{-4}{5}}$

$h(x)=\frac{1}{2250}^{3}-\frac{2}{15}x^2+\frac{176}{15}x+\frac{2920}{9}$

L2 (x)=-1.6x+120



Conclusion:

We were able to find the formulas needed to construct the correct slopes of the roller coaster and to create smooth transitions.

References:

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"roller coaster." *Collins English Dictionary - Complete & Unabridged 10th Edition*. HarperCollins Publishers. <Dictionary.com http://dictionary.reference.com/browse/roller coaster>

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