Where Should A Pilot Start Descent?

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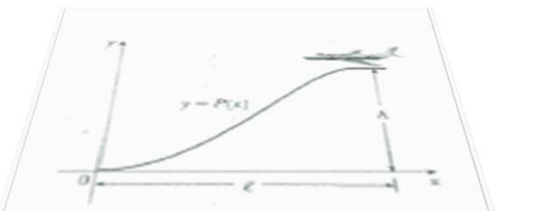
MAT1475H: Calculus I Honors, Mathematics Department
New York City College of Technology, CUNY- Spring 2014

Research Question

Where should a pilot start descent?



http://www.stewartcalculus.com/data/CALCULUS%207E/upfiles/projects/ess_wp_0205_stu.pdf

Introduction

We will use our understanding of basic calculus to model how a plane lands. With the help of modern research and scientific calculation, we will be able to compute the phases of which plane goes through while it is landing. This question is adapted from Stewart, *Single Variable Calculus: Early Transcendentals*.

Information we know

Things we will have to know/consider to solve:

- While descending, the pilot must maintain a constant horizontal speed v throughout the decent.
- The cruising Altitude is h when descent starts at a horizontal distance l from touching down at the origin.
- We will have to assume into some physical disadvantages such as motion sickness. To avoid that, the absolute value of the vertical acceleration should not exceed a constant k. (which is much less than the acceleration due to gravity.)

References

- Calculus Early Transcendentals , Single variable, Second Edition, Author: Jon Rogawski

1) Find a cubic polynomial $P(x) = ax^3+bx^2+cx+d$ that satisfies conditions 1 by imposing suitable conditions on P(x) and P'(x) at the start of descent and at touchdown?

The original equation is $P(x) = ax^3 + bx^2 + cx + d$

P (0) = 0, so plug in zero for x. You will find that d = 0.

P (0) =0 Altitude is zero, when plane is on ground.

$$ax^3 + bx^2 + cx + d$$
$$a(0)^3+b(0)^2+c(0)+d=0$$
$$d=0$$

Simplify d=0 into $ax^3 + bx^2 + cx + d$, which is $ax^3 + bx^2 + cx$

Find the derivative of P(x), which is $3ax^2+ 2bx + c$

Simplify the equation to $P(x) = 3ax^2+ 2bx + c$

$$P(0) = 3a(0)^2+2b(0)+c$$
$$c=0$$
$$P(x) = ax^3+bx^2$$

$P(l)=h$ at a certain length, the altitude is h.

$P(l)=al^3+bl^2=h$

Take derivative $P(l)= al^3+bl^2$, then simplify for b

$$3al^2+2bl=0$$
$$3al^2= -2bl$$
$$b= (-3al)/2$$

Now plug b value into the original equation

$$al^3+ ((-3al)/2)l^2=h$$

Solve for a:

$$a = -2h/l^3$$

Now put a value in b to find the exact value:

$$b= (-3(-2h/l^3)*l)/2$$
$$= 3h/l^2$$

2) Use condition 2 and 3 to show that $6hv^2/l^2 \leq k$

$y=P(l)=al^3+bl^2$ (note: x→l in this equation)

$dy/dt=P'(l)=3al^2(dl/dt)+2bl(dl/dt)$ (note: dl/dt= -v)

$$P'(l)=3al^2(-v)+2bl(-v) = -3avl^2-2bvl$$

$d^2y/dt^2= -6avl(dl/dt)-2bv(dl/dt)$

$$= -6avl(-v)-2bv(-v)$$
$$= -6av^2l-2bv^2 \text{ (note: } d^2y/dt^2 \leq k)$$
$$-6Lav^2 -2bv^2 \leq K$$

Now plug a and b value:

$$-6lv^2(-2h/l^3) - 2(3h/l^2)v^2 \leq K$$
$$12hlv^2/l^3 – 6hv^2/l^3 \leq K$$
$$12hv^2/l^2 - 6hv^2/l^2 \leq K$$
$$6hv^2/l^2 \leq k$$

3) Suppose that an airline decides not to allow vertical acceleration of a plane to exceed k = 860 mi/h². If the cruising altitude of a plane is 35,00 ft, and the speed is 300 mi how far away from the airport should the pilot start descent?

$6hv^2/l^2 \leq k$

Change from feet to miles

$(6(6.629)*(300)2)/l^2 \leq 860$

(35000ft)(1mi/5280ft)*

Solve for l: =6.629mi

l 64.5 mi

Let's assume l = 64.5

Graph

$A= -2h/l^3$

$h= 6.629$, $l= 64.5$

$A= (-2*6.629) / (64.5)^3$

$A= -.00004937$

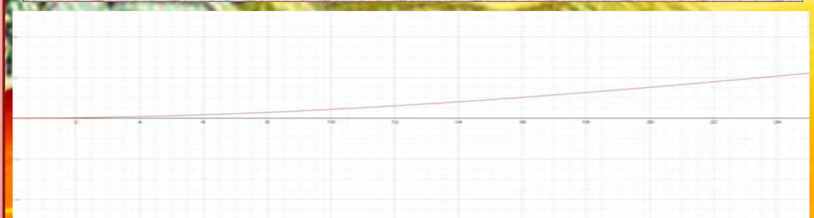
$B= 3h/l^2$

$h= 6.629$, $l= 64.5$

$B= (3*6.629) / (64.5)^2$

$B= 0.004777$

$P(x) = ax^3+bx^2$ $-.00004937x^3+0.004777x^2$



Conclusion

With calculus, we were able to compute real life problems that we face in our everyday situations. Understanding the theoretical part of this has made us aware that the plane has to be reached at an approximate value(64.5mi) in order to successfully descend.