

MAT 1475H
Spring 2014
Professor K. Poirier
Final Exam
May 21, 2014

Name (Print): _____

Time Limit: 100 Minutes

This exam contains 7 pages and 9 problems. Check to see if any pages are missing. Print your name on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may use a calculator on this test. No other aids are allowed.

1. Evaluate the following limits. Explain your answers.

(a) (5 points)

$$\lim_{x \rightarrow 0} \frac{x - 1}{x^2 - 1}$$

(b) (5 points)

$$\lim_{x \rightarrow 1} \frac{x - 1}{x^2 - 1}$$

(c) (5 points)

$$\lim_{x \rightarrow -1^+} \frac{x - 1}{x^2 - 1}$$

(d) (5 points)

$$\lim_{x \rightarrow -\infty} \frac{x - 1}{x^2 - 1}$$

2. Let

$$f(x) = 2x^3 + 4x + 5$$

(a) (5 points) Use the *limit definition* of the derivative to differentiate $f(x)$.

(b) (5 points) Find the *equation* of the tangent line to $f(x)$ at $x = -1$. Leave your answer in $y = mx + b$ form.

3. Differentiate the following functions. Do not simplify your answers.

(a) (5 points)

$$f(x) = (x^2 + x + 1) \left(\frac{4x^3 - 2x}{6x^5 + 7x^2} \right)$$

(b) (5 points)

$$f(x) = 7e^{x \cos(2x)}$$

4. (5 points) Use linearization to approximate $\sqrt[3]{27.3}$.

5. (5 points) Find $\frac{dy}{dx}$.

$$2xy^3 + 3xy = 15$$

6. (5 points) Usain Bolt is a Jamaican sprinter who is widely recognized as the fastest man on Earth. Imagine that you are operating a video camera to capture his world-record breaking finish in a race. You and your camera are positioned along the finish line, 10 meters from where Bolt will cross it. To keep Bolt in the frame as he runs the race, you must turn the camera. If the camera is turning at a rate of 1 radian per second, determine Bolt's speed when he crosses the finish line. (Hint: draw a picture!)

7. (5 points) The United States Post Office defines a parcel as *oversized* if the sum of its height and the perimeter of its base is greater than 108 inches. Find the dimensions of a box with square base that is not oversized and has maximum volume.

8. Let

$$f(x) = xe^{-x^2}$$

- (a) (5 points) Determine the intervals of increase and decrease for $f(x)$.
- (b) (2 points) Determine the x coordinates of any relative maxima and minima for $f(x)$.
- (c) (5 points) Determine the intervals where the graph of $f(x)$ is concave up and where $f(x)$ is concave down.
- (d) (3 points) Determine the x coordinates of any points of inflection.

9. (5 points) Evaluate the definite integral:

$$\int_1^2 \left(\sin(2\pi x) + x^2 + \frac{1}{x} \right) dx$$