MAT 2440 **Name (Print):** ____________________

Spring 2017

Professor K. Poirier

Final Exam **Total: 60 points**

May 25 **Time Limit: 100 Minutes**

This exam contains 9 pages and 8 problems, each with multiple parts, plus one extra credit problem. Each page contains all the parts for one problem. Complete each part of 6 of the 8 problems. If you attempt more than 6 of the problems, only the first 6 will be graded.

Check to see if any pages are missing. Print your name on the top of this page, and put your initials on the top of every page, in case the pages become separated. You may use a calculator on this test. No other aids are allowed. Show all your work for full credit.

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1. (10 points) (a) Let p represent the proposition "The election is decided," and let q represent "The votes have been counted." Express the following compound proposition as an English sentence.

   $\neg q \vee (\neg p \wedge q)$

   (b) Let $C(x)$ represent "x is a comedian" and let $F(x)$ represent "x is funny," where the domain consists of all people. Express the following as an English sentence with no variables.

   $\forall x(C(x) \to F(x))$

   (c) Let $M(x, y)$ repesent "x has sent y an e-mail message" and let $T(x, y)$ represent "x has telephoned y," where the domain for each variable consists of all students in your class. Use quantifiers to express the following statement as a logical proposition. (Assume that messages that were sent are received.)

   Every student in the class has either received an email message or received a telephone call from another student in the class.

2. (10 points) (a) Determine whether the following argument is valid. If the argument is valid, determine the rule of inference being used. If the argument is not valid, determine the logical error that occurs.

   If n is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then $n > 1$.

   (b) Prove or disprove that the product of any two irrational numbers is always irrational.

3. (10 points) (a) Determine the cardinality of each of these sets.

(i) $\{a\}$

(ii) $\{\{a\}\}$

(b) Assume that A and B are sets with $A \subseteq B$.

(i) Show that $A \cup B = B$.

(ii) Show that $A \cap B = A$.

(c) Let $a_0, a_1, \ldots, a_n$ be a sequence of real numbers. Prove that $\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0$.

4. (10 points) (a) Define the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ by $f(m, n) = m^2 + n^2$. Determine whether the function f is onto or not onto. Explain your answer.

(b) Determine whether each of the following sets is finite, countably infinite, or uncountable. Circle the correct answer. You do not need to explain your answer.

the negative integers	**finite**	**countably infinite**	**uncountable**
the even integers	**finite**	**countably infinite**	**uncountable**
the integers less than 100	**finite**	**countably infinite**	**uncountable**
the real numbers between 0 and 1	**finite**	**countably infinite**	**uncountable**
the positive integers less than 1,000,000,000	**finite**	**countably infinite**	**uncountable**
the integers that are multiples of 7	**finite**	**countably infinite**	**uncountable**

(c) Encrypt the message DO NOT PASS GO by translating the letters into numbers, applying the given encryption function, and then translating the numbers back into letters.

$$f(p) = (3p + 7) \mod 26$$

5. (10 points) (a) Consider the following mystery algorithm.

procedure *mystery*($a_1, a_2, ..., a_n$: integers)
$m := a_1$
for $i := 2$ to n
    if $m < a_i$ then $m := a_i$
return m

In plain English, describe what the output of this algorithm is when the input is the list $a_1, a_2, ..., a_n$.

List all the steps used by the mystery algorithm when the input is the list 1, 8, 12, 9, 11, 2, 14, 5, 10, 4 and determine the output.

(b) Use pseudocode to give recursive algorithm for finding the sum of the first n odd positive integers.

(c) Prove that your algorithm in part (b) is correct.

6. (10 points) (a) List five integers that are congruent to 4 modulo 12.

(b) Convert the binary expansion of this integer to a decimal expansion $(111\ 1100\ 0001\ 1111)_2$

(c) Find the greatest common divisor and the least common multiple of 1000 and 625. Verify that $\gcd(1000, 625) \cdot \text{lcm}(1000, 625) = 1000 \cdot 625$.

(d) Determine the first five terms of the sequence of pseudorandom numbers that is generated using the linear congruential generator $x_{n+1} = (4x_n + 1) \mod 7$ with seed $x_0 = 3$.

7. (10 points) (a) Prove that $3+3\cdot 5+3\cdot 5^2+\cdots+3\cdot 5^n = \frac{3(5^{n+1}-1)}{4}$ whenever n is a nonnegative integer.

(b) A jigsaw puzzle is put together by successively joining pieces that fit together into blocks. A *move* is made each time a piece is added to a block, or when two blocks are joined. Prove that no matter how the moves are carried out, exactly $n-1$ moves are required to assemble a puzzle with n pieces.

8. (10 points) (a) Build a binary search tree for the words *banana, peach, apple, pear, coconut, mango,* and *papaya* using alphabetical order.

   (b) You have 6 coins, one of which is counterfeit. The counterfeit coin is identical to the other coins except that it is either lighter or heaver than the others. You have a balance scale to determine which coin is counterfeit and whether it is lighter or heavier than the others. Build a decision tree that describes an algorithm to determine which coin is counterfeit and whether it is lighter or heavier than the others.

Extra credit. Solve this problem only if you have completed 6 of the above problems. This problem is worth a possible 2 extra points only and there is no partial credit.

Prove or disprove:

There exists a ten digit number where the first digit is the number of times a zero appears in the number. The second digit is the number of times a one appears in the number. The third digit is the number of times a two appears in the number. This pattern continues until the last digit is the number of times a nine appears in the number.

If there exists such a number, find it. If there does not exist such a number, explain why not.