Chapter 5. Quantum Gates

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1 1-qubit Unitary Gates

In quantum computing the unitary operator U acting on a single-qubit is referred as a1-qubit gate. We can represent 1-qubit gate operator on the 2-dimensional Hilbert space of a single qubit as 2×2 matrices. These linear operators are specified completely by their action on the computational basis qubits. Every one qubit pure state is represented as a unit vector whose origin is fixed at the center of the Bloch sphere or equivalently as a point on the surface of the Bloch sphere. 1-qubit quantum gate operator transforms a quantum state $|\psi\rangle$ into he other quantum state $\left|\psi^{'}\right\rangle$ as $\left|\psi^{'}\right\rangle = U|\psi\rangle$. It terms of the Bloch sphere, the action of operator U of the state $|\psi\rangle$ can be thought of as a rotation of the state vector for $|\psi\rangle$ to the state vector for $\left|\psi^{'}\right\rangle$. In terms of the Bloch sphere, this action can be visualized as a rotation through the angles about the x and z axes.

1.1 X Gate

Any unitary operator acting on a two-dimensional quantum system that is a qubit is called 1-qubit quantum gate. Any quantum gate A transforms a quantum state $|\psi\rangle$ into another quantum state $A|\psi\rangle$. X gate is the quantum equivalent of a classical NOT gate. Since the states of a qubit $|0\rangle$ and $|1\rangle$

$$X|0\rangle = |1\rangle \text{ or } X|1\rangle = |0\rangle \tag{1}$$

X gate is also called *qubit-flip* gate because it inverts each input qubit. Representing a qubit by a column vector one can find the matrix form of the X gate

$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ or } X|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{2}$$

The matrix of the X gate, $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, is the same as Pauli matrix σ_x it is called X gate because of this reason. If two X gates are connected in series to form a quantum circuit then the first X gate transforms the quantum system $|\psi\rangle$ to $\left|\psi^{'}\right\rangle = X|\psi\rangle$ and then the second X gate acts on it to form $X\left|\psi^{'}\right\rangle = XX|\psi\rangle = |\psi\rangle$. Therefore, the output is the same as the original input and two X gates connected in series act as an identity matrix I :

$$XX = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \equiv I. \tag{3}$$

Let's consider the state of superposed qubit: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. When a superposed state goes through the X gate the result is

$$X|\psi\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} (\alpha|0\rangle + \beta|1\rangle) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha|1\rangle + \beta|0\rangle. \tag{4}$$

Thus, X gate "negates" the computational basis states $|0\rangle$ and $|1\rangle$ correctly, but cannot correctly negate an arbitrary superposition state. Using Dirac notation X gate can be presented as

$$X = |0\rangle\langle 1| + |1\rangle\langle 0|, \text{ therefore } X|\psi\rangle = (|0\rangle\langle 1| + |1\rangle\langle 0|)(\alpha|0\rangle + \beta|1\rangle) = \alpha|1\rangle + \beta|0\rangle. \tag{5}$$

1.2 Y Gate

The gate represented by the Pauli matrix σ_y is called Y gate. This gate maps $|0\rangle$ computational basis state to the state $i|1\rangle$, and $|1\rangle$ computational basis state to the state $-i|0\rangle$:

$$Y|0\rangle = i|1\rangle \text{ or } Y|1\rangle = -i|0\rangle \tag{6}$$

Representing a computational basis state in the vector form one can find the matrix form of the Y gate

$$Y|0\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = i \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ or } Y|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -i \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{7}$$

Therefore, Y gate define the following transformation of a superposed qubit

$$Y|\psi\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} (\alpha|0\rangle + \beta|1\rangle) = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = i\alpha|1\rangle - i\beta|0\rangle. \tag{8}$$

Using Dirac notation Y gate can be presented as

$$Y = i|1\rangle\langle 0| - i|0\rangle\langle 1|, \text{ therefore } Y|\psi\rangle = (i|1\rangle\langle 0| - i|0\rangle\langle 1|)(\alpha|0\rangle + \beta|1\rangle) = i\alpha|1\rangle - i\beta|0\rangle. \tag{9}$$

It is important to mention that $YY = Y^2 = I$ is the identity matrix.

1.3 Z Gate

Z gate maps input $|j\rangle$ to the state $(-1)^j |j\rangle$. For computational basis states $|1\rangle$ and $|0\rangle$ written in the vector form one can find the matrix form of the Z gate

$$Z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ or } Z|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -\begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{10}$$

Thus, Z gate maps the superposed qubit $|\psi\rangle$ as

$$Z|\psi\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} (\alpha|0\rangle + \beta|1\rangle) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha|0\rangle - \beta|1\rangle. \tag{11}$$

Due to this mathematical structures this transformation is called a *phase-flip*. Using Dirac notation Z gate can be presented as

$$Z = |0\rangle\langle 0| - |1\rangle\langle 1|, \text{ therefore } Z|\psi\rangle = (|0\rangle\langle 0| - |1\rangle\langle 1|)(\alpha|0\rangle + \beta|1\rangle) = \alpha|1\rangle - \beta|0\rangle. \tag{12}$$

It is important to mention that $ZZ = Z^2 = I$ is the identity matrix. Therefore, for X, Y, and Z gates we have $X^2 = Y^2 = Z^2 = I$.

1.4 Square-Root of NOT Gate

Square-Root of NOT gate is a 1-qubit quantum gate presented by an operator $\sqrt{X}$ which acts on a two-dimensional quantum system. This 1-qubit quantum gate is designed to implement the expression:

$$\sqrt{X} \cdot \sqrt{X} = X \tag{13}$$

There is no such gate in classic logic. This gate is an example of how a gate can exist even though Boolean algebra cannot be used to describe this operation. The $\sqrt{X}$ gate can be represented by the following matrix

$$\sqrt{X} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \tag{14}$$

When $\sqrt{X}$ is applied to the computational basis states $|0\rangle$ and $|1\rangle$, the output is

$$\sqrt{X}|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right), \tag{15}$$

$$\sqrt{X}|1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = -\frac{1}{\sqrt{2}} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right), \tag{16}$$

Therefore from above expressions one can conclude that when a NOT gate $\sqrt{X}$ is applied to $|0\rangle$ or $|1\rangle$ state it leaves the qubit in an equal superposition of states $|0\rangle$ and $|1\rangle$. The square-root NOT gate $\sqrt{X}$ transforms a superposed qubit as

$$\sqrt{X}\,|\psi\rangle = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} (\alpha\,|0\rangle + \beta\,|1\rangle) = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha\,|0\rangle - \beta\,|1\rangle\,. \quad (17)$$

In Dirac notation $\sqrt{X}$ gate can be presented as

$$\begin{aligned}
\sqrt{X} &= \frac{1}{\sqrt{2}}\,(|0\rangle\,\langle 0| - |0\rangle\,\langle 1| + |1\rangle\,\langle 0| + |1\rangle\,\langle 1|)\,, \text{ therefore} \\
\sqrt{X}\,|\psi\rangle &= \frac{1}{\sqrt{2}}(|0\rangle\,\langle 0| - |0\rangle\,\langle 1| + |1\rangle\,\langle 0| + |1\rangle\,\langle 1|)(\alpha\,|0\rangle + \beta\,|1\rangle) \\
&= \frac{1}{\sqrt{2}}\,(\alpha\,|0\rangle - \beta\,|1\rangle + \alpha\,|1\rangle + \beta\,|1\rangle)\,. \quad (18)
\end{aligned}$$

$\sqrt{X}\sqrt{X}$ transforms a superposed qubit $|\psi\rangle\,\alpha\,|0\rangle + \beta\,|1\rangle$ as

$$\begin{aligned}
\sqrt{X}\sqrt{X}\,|\psi\rangle &= \sqrt{X}\frac{1}{\sqrt{2}}\,(\alpha\,|0\rangle - \beta\,|1\rangle + \alpha\,|1\rangle + \beta\,|1\rangle) \\
&= \frac{1}{\sqrt{2}}(|0\rangle\,\langle 0| - |0\rangle\,\langle 1| + |1\rangle\,\langle 0| + |1\rangle\,\langle 1|)\frac{1}{\sqrt{2}}\,(\alpha\,|0\rangle - \beta\,|1\rangle + \alpha\,|1\rangle + \beta\,|1\rangle) \\
&= \frac{1}{2}\,(\alpha\,|0\rangle + \beta\,|1\rangle + \alpha\,|0\rangle + \beta\,|1\rangle) = |\psi\rangle\,. \quad (19)
\end{aligned}$$

1.5 Hadamad Gate

One of the most important gate in quantum computing is the Hadamad gate, which is truly quantum gate. The Hadamad gate H maps input $|k\rangle$ to

$$\begin{aligned}
H\,|k\rangle &= \frac{1}{\sqrt{2}}\sum_{m=0,1}(-1)^{km}\,|m\rangle + \beta\,|1\rangle) = \\
& \frac{1}{\sqrt{2}}\,\left(|0\rangle + (-1)^k\,|1\rangle\right)\,. \quad (20)
\end{aligned}$$

The Hadamad gate acts on the computational basis states $|0\rangle$ and $|1\rangle$ as follows:

$$\begin{aligned}
H|0\rangle &= H\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\left(|0\rangle + (-1)^0|1\rangle\right) \\
&= \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right); \qquad (21) \\
H|1\rangle &= H\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\left(|0\rangle + (-1)^1|1\rangle\right) \\
&= \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right). \qquad (22)
\end{aligned}$$

Hence, one can write the Hadamad gate in the matrix form as

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \qquad (23)$$

while using the Dirac notation H gate corresponds to the operator

$$H = \frac{1}{\sqrt{2}}\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|\right). \qquad (24)$$

The Hadamad gate acts on the superposed state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ the following way

$$\begin{aligned}
H|\psi\rangle &= \frac{1}{\sqrt{2}}\left(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|\right)\left(\alpha|0\rangle + \beta|1\rangle\right) \\
&= \alpha\frac{|0\rangle + |1\rangle}{\sqrt{2}} + \beta\frac{|0\rangle - |1\rangle}{\sqrt{2}} = \\
&\frac{\alpha+\beta}{\sqrt{2}}|0\rangle + \frac{\alpha-\beta}{\sqrt{2}}|1\rangle \qquad (25)
\end{aligned}$$

The Hadamad gate converts the computational basis states $|0\rangle$ or $|1\rangle$ to a superposition state of $|0\rangle$ and $|1\rangle$. When superposed qubit is measured there is an equal probability of it being in the state $|0\rangle$ or $|1\rangle$.

1.6 S Phase Gate

S phase gate is represented by the following matrix:

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}, \qquad (26)$$

and this gate turns the computational basis states $|0\rangle$ and $|1\rangle$ into $|0\rangle$ and $i|1\rangle$, respectively, as shown below

$$\begin{aligned}
S|0\rangle &= \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle, \qquad (27) \\
S|1\rangle &= \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle. \qquad (28)
\end{aligned}$$

S gate corresponds to the operator $S = |0\rangle\langle 0| + i\,|1\rangle\langle 1|$. The S gate transforms the superposed state $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ in the following way

$$S\,|\psi\rangle = S\,(\alpha\,|0\rangle + \beta\,|1\rangle) = \alpha\,|0\rangle + i\beta\,|1\rangle\,. \tag{29}$$

The action of two S gates in serios is equivalent to applicacion of a Z gate

$$S^2\,|\psi\rangle = SS\,|\psi\rangle = S(\alpha\,|0\rangle + i\beta\,|1\rangle) = \alpha\,|0\rangle - \beta\,|1\rangle \equiv Z\,|\psi\rangle \tag{30}$$

which follows from Eq. zgate. In a matrix form

$$S^2 = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & \\ 0 & -1 & \equiv Z \end{bmatrix}, \tag{31}$$

1.7 T Gate

T gate rotates the computational basis state $|1\rangle$ in 2D Hilbert space by the angle $\frac{\pi}{8}$, not effect on the state $|0\rangle$ and is represented by the matrix

$$T = \begin{bmatrix} 1 & 0 \\ 0 & \exp\left(\frac{i\pi}{8}\right) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1+i}{\sqrt{2}} \end{bmatrix}, \tag{32}$$

while the correspondinf Dirac operator is

$$T = |0\rangle\langle 0| + \frac{1+i}{\sqrt{2}}\,|1\rangle\langle 1|\,. \tag{33}$$

Thus, the T gate difines the transformation of the computational basis states $|0\rangle$ and $|1\rangle$, and the superposed state $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ as

$$T\,|0\rangle = |0\rangle \tag{34}$$

$$T\,|1\rangle = \frac{1+i}{\sqrt{2}}\,|1\rangle \tag{35}$$

$$T\,|\psi\rangle = \alpha\,|0\rangle + \beta\frac{1+i}{\sqrt{2}}\,|1\rangle \tag{36}$$

Interestigly eaugh a conection two T gates in series forms a S gate, $T^2 = S$:

$$\begin{aligned} T^2 &= TT = T\left(|0\rangle\langle 0| + \frac{1+i}{\sqrt{2}}\,|1\rangle\langle 1|\right) \\ &= \left(|0\rangle\langle 0| + \frac{1+i}{\sqrt{2}}\,|1\rangle\langle 1|\right)\left(|0\rangle\langle 0| + \frac{1+i}{\sqrt{2}}\,|1\rangle\langle 1|\right) = |0\rangle\langle 0| + i\,|1\rangle\langle 1| \equiv S \end{aligned} \tag{37}$$

1.8 $R(\theta)$ Rotational Gate

A rotational gates represent very important class of 1-qubit gates. If we exponentiate the X, Y and Z gates we get unitary operators corresponding to rotations about the x-, y, and z-axes of the Bloch sphere. The rotational gates are defined as follows:

$$R_x(\theta) = \exp\left(-\frac{\theta X}{2}\right) = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}X, \tag{38}$$

$$R_y(\theta) = \exp\left(-\frac{\theta Y}{2}\right) = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}Y, \tag{39}$$

$$R_z(\theta) = \exp\left(-\frac{\theta Z}{2}\right) = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}Z. \tag{40}$$

Knowing the identity matrix I and matrices X, Y, and Z we can write the rotatinal gates in the computational basis:

$$R_x(\theta) = \begin{bmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}, \tag{41}$$

$$R_y(\theta) = \begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}, \tag{42}$$

$$R_z(\theta) = \begin{bmatrix} \cos\frac{\theta}{2} - i\sin\frac{\theta}{2} & 0 \\ 0 & \cos\frac{\theta}{2} + i\sin\frac{\theta}{2} \end{bmatrix}, \tag{43}$$

The R gates define the transformation of the computational basis states $|0\rangle$ and $|1\rangle$ as

$$R_x(\theta)|0\rangle = \begin{bmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \cos\frac{\theta}{2}|0\rangle - i\sin\frac{\theta}{2}|1\rangle, \tag{44}$$

$$R_x(\theta)|1\rangle = \begin{bmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = -i\sin\frac{\theta}{2}|0\rangle + \cos\frac{\theta}{2}|1\rangle; \tag{45}$$

$$R_y(\theta)|0\rangle = \begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}|1\rangle, \tag{46}$$

$$R_y(\theta)|1\rangle = \begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = -\sin\frac{\theta}{2}|0\rangle + \cos\frac{\theta}{2}|1\rangle; \tag{47}$$

$$
\begin{aligned}
R_z(\theta)\,|0\rangle &= \begin{bmatrix} \cos\frac{\theta}{2} - i\sin\frac{\theta}{2} & 0 \\ 0 & \cos\frac{\theta}{2} + i\sin\frac{\theta}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
&= \left(\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\right)|0\rangle = \exp\left(-\frac{\theta}{2}\right)|0\rangle\,, \qquad (48) \\
R_z(\theta)\,|1\rangle &= \begin{bmatrix} \cos\frac{\theta}{2} - i\sin\frac{\theta}{2} & 0 \\ 0 & \cos\frac{\theta}{2} + i\sin\frac{\theta}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
&= \left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)|0\rangle = \exp\left(\frac{\theta}{2}\right)|1\rangle\,. \qquad (49)
\end{aligned}
$$

The graphical representation of 1-qubit quantum gates is given in Fig. 1.

2 Reversible Logic and Controlled Gates

In classical computations in the reversible gate it is possible to unambiguosly determine bits on the inputs of the gate from the outputs of the gate. Classical gates are irreversible: all possible input combinations in such a gate are mapped to one of two possible outputs. Fot example, in twoinput AND gates the possible input combinations are 00, 01, 10, and 11. The combintions 00, 01, 10 map to output 0, and oly the combination 11 produces an ouput 1. Therefore, when an AND gate produces 0, the inpu could be one of the three possibilities and it is not possible to infer what the actual input was. Since, in an AND gate a complite knowleadge about the input cannot be determine from the output, the operation of the AND is not reversible. You can check in similar manner all other classical gate by analyzing the truth Table and see that the input values cannot be determined from the output value. Only the NOT gate is an exception. For this gate if one knows output the input can be determined definitely. The explanation is the following: since in classical gates such as two-input gates AND, OR, NAND, NOR a 2-bit input maps into a 1-bit output, one-bit information is lost on every operation and it is impossible to recover. These classical gates are irreversable. Hence, a reversable gate must have exacly the same number of inputs and outputs. This will allow the input value to be uniquely determined from the output value and vice versa. This is a reason a NOT gate with the single input and a single output is reversable. In quantum computing circuits is an extremely useful reverable gate with two-inputs/two-outputs which is known as controlled-gate and controlled-NOT gate known as CNOT is an example.

2.1 CNOT Gate

A CNOT gate can be used to generate entanglement and this gate graphicall represented in Fig. 2. Here a is a controlled input and the dependence of output y is shown by a vertical line from a to one of the output. The input a is typically called the sourse and the input b is known as the target

a	b	x	y
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

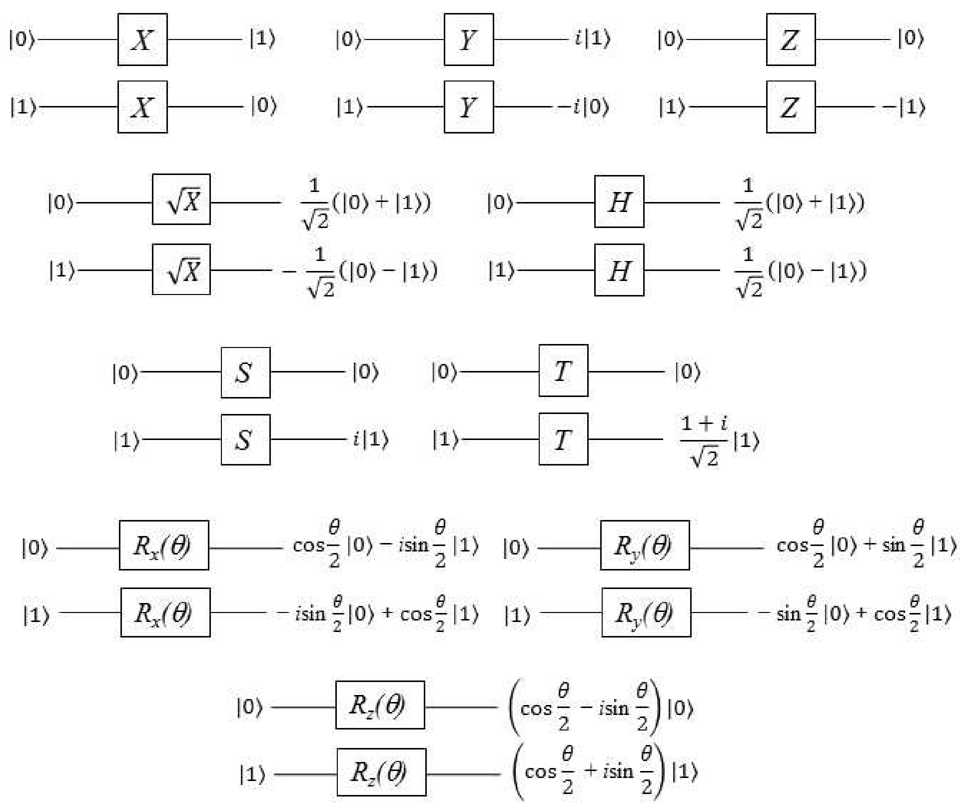

Figure 1: 1-qubit gates