1) a) $(x^2-9)y'' - 2y' - 4y = 0$

$P_0(x)y'' + P_1(x)y' + P_2(x)y = 0$

$P_0(x) = x^2 - 9$

$x = \pm 3$

$\rho = |3-0|$

$\rho = 0 \quad (-3,3)$

b) $y = \sum_{k=0}^{\infty} c_k x^k$

$y' = \sum_{k=0}^{\infty} k c_k x^{k-1}$

$y'' = \sum_{k=0}^{\infty} k(k-1) c_k x^{k-2}$

$15 = (x^2-9)y'' - 2y' - 4y$

$= (x^2-9) \sum_{k=0}^{\infty} k(k-1) c_k x^{k-2} - 2\sum_{k=0}^{\infty} k c_k x^{k-1} - 4 \sum_{k=0}^{\infty} c_k x_k$

$= \sum_{k=0}^{\infty} k(k-1) c_k x^k - \sum_{k=0}^{\infty} k c_k x^{k-2} - \sum_{k=0} 2k c_k x^{k-1} - \sum_{k=0}^{\infty} 4 c_k x^k$

(A) (B) (C) (D)

$\sum_{k=0}^{\infty} 9k(k-1) c_k x^{k-2} = 9(0)(0-1) c_0 x^{k-2} + 9(1)(1-1) c_1 x^{1-2} + \sum_{k=3}^{\infty} 9k(k-1) c_k x^{k-2}$

$\underset{\text{shift by 2}}{=} \sum_{k=0}^{\infty} 9(k+2)(k+1) c_k x^k$

c) $\sum_{k=0}^{\infty} 2k c_k x^{k-1} = 2(0) c_0 x^{k-1} + \sum_{k=1}^{\infty} 2k c_k x^{k-1} \underset{\text{shift by}}{=} \sum_{k=0}^{\infty} 2(k+1) c_k x^k$

plug back in

$15 = \sum_{k=0}^{\infty} \left[-9(k+2)(k+1) c_{k+2} - 2(k+1) c_k + 1 + [k(k-1) - 4] c_k \right] x^k = 0$

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