Project #3: Hydroplane Racing

Throughout history many engineers have gathered research information on how to reduce the drag in vehicles like cars, boats, planes, etc., but in hydroplane racing it isn't only the drag that wind creates that engineers must focus on but also the drag that water creates. Hydroplane racing has been around since the 1900's but many of the boats that started it were incapable of reaching higher speeds due to the design of the boat and the condition of the water. It wasn't until 1910 when the first "step" hydroplanes were introduced; a "step" is a notch that is located on the underside of the hull, which allowed the boats to plane over the water with less friction. With this new design many boats were capable of hitting higher speeds but this created another problem; despite the new "step" design allowing boats to hit higher speeds the "step" design also made it hard for racers to control the boat. A few years went by and a new design was introduced in the late 1930's, this design was called "The Three Point Hydroplane", which took the "step" of the hydroplane and split it in two then connecting them on the opposite sides of the hull.

These new parts were called sponsons and increased the area of the boat which allowed it to have better control and since the boat was now wider it decreased the boats chances of tipping over. However, both designs had its advantages and disadvantages, for example despite the three point hydroplanes improving the boats handling these boats are incapable of racing in the ocean due to how light and balanced they were. Unlike the "step" boats where their underbody helped them cut through the water instead of bouncing on it, this meant that three point hydroplanes were only capable of racing in smaller bodies of water and "step" hydroplanes were capable of racing in all bodies of water. In this report we will mainly concentrate on the drag created by the water since that is the biggest factor when racing these hydroplanes and the terminal velocity. The reason I chose this topic is because I enjoy all types of racing and learning about the differences between all vehicles. I enjoy doing research on how to improve a vehicle's performance as well as how the environment affects it.

Real world problem: Hydroplane boats travel at very high speeds with the use of a very powerful engine, on average most hydroplane boats have around 1000+ horsepower. With this amount of power being produced these boats need a solution in order to prevent drag created by the water. The reason for this is because when it comes to racing, regardless of the vehicles being used, many teams want to create a vehicle that can overcome any type of drag; that is why many engineers focus on the aerodynamics of the vehicles. Without the proper structure many vehicles would not be capable of reaching their full potential and when it comes to racing every second counts as each racer is trying to be the fastest in the league; while also trying to set new records.

Differential equations:

Drag Force:

Similar to friction, drag force acts in a direction opposite to the motion of an object. However, unlike friction, the magnitude of the drag force is determined by a complex function of the object's velocity within a fluid and is dependent on factors such as the object's size, shape, velocity, and the characteristics of the fluid. For most large objects that are not moving too slowly, like bicycles, cars, and baseballs, the drag force is proportional to the square of the object's velocity, expressed mathematically as FD∝v2. However, when considering other factors, this relationship becomes:

$$F_D = \frac{1}{2} C \rho A v^2$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid.

Terminal Velocity:

Newton's second law is also evident when considering hydroplane boats. As a hydroplane boat moves through the water, it experiences both a forward force from its engine and a drag force from the water acting against its hull. As the boat accelerates, the magnitude of the drag force increases until it equals the forward force from the engine. At this point, the boat's net force is zero, meaning there is no further acceleration and the boat has reached its terminal velocity. Because the drag force is proportional to the square of the boat's speed, a heavier boat must go faster to reach its terminal velocity compared to a lighter boat with the same engine power.

At terminal velocity:

$$F_{net} = mg - F_D = ma = 0$$

Thus,

$$mg = F_D$$

Using the equation for drag force we get,

$$mg = \frac{1}{2} C \rho A v_T^2$$

To solve for velocity we have,

$$v_T = \sqrt{\frac{2mg}{\rho C A}}$$

The Calculus of Velocity Dependent Frictional Forces:

The frictional force on an object sliding across a surface can be approximated by a constant value of μkN. However, when an object moves through a fluid like gas or liquid, the drag force acting on it is usually not as straightforward. This force is a complex function of the object's velocity, size, shape, and the properties of the fluid it is moving through. However, for a body moving in a straight line at moderate speeds through a liquid such as water, the frictional force can often be approximated by

$$F_R = - bv$$

For example, when a motorboat is moving across a lake at a speed of v_0, when its motor suddenly freezes up and stops. The boat then slows down under the frictional force $F_R = - bv$ What are the velocity and position of the boat as functions of time?

With the motor stopped, the only horizontal force on the boat is $F_R = - bv$, so from Newton's second law,

$$m\frac{dv}{dt} = - bv$$

Which can be written as,

$$\frac{dv}{v} = - \frac{b}{m}dt$$

Integrating this equation between the time zero when the velocity is v0 and the time t when the velocity is v,

$$\int_0^v \frac{dv'}{v'} = - \frac{b}{m}\int_0^t dt'$$

Turns into,

$$ln\frac{v}{v_0} = - \frac{b}{m}t$$

Since lnA = x implies e^x = A, we can write this as,

$$v = v_0 e^{-\frac{bt}{m}}$$

From the definition of velocity,

$$\frac{dx}{dt} = v_0 e^{-\frac{bt}{m}}$$

Which turns into,

$$dx = v_0 e^{-\frac{bt}{m}} dt$$

Since the initial position is zero, we have

$$\int_0^x dx' = v_0 \int_0^t e^{-\frac{bt'}{m}} dt'$$

Which then turns into,

$$x = -\frac{mv_0}{b} e^{-\frac{bt}{m}} \Big|_0^t = \frac{mv_0}{b}\left(1 - e^{-\frac{bt}{m}}\right)$$

As the time increases, $e^{-\frac{bt}{m}} \rightarrow 0$, and the position of the boat reaches a limiting value

$$x_{max} = \frac{mv_0}{b}$$

Using this example we can find the limiting values. In which terminal velocity is the same as limiting velocity, using this boat example we found that limiting distance is also similar. Since they are similar, the limiting distance will let us know how far the boat will travel after a certain amount of time has passed. Using the properties of exponential decay, the time involved to reach these values is actually not too long but they are easily found by taking the limit to infinity.

References:

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