Test #3 Version B

Find the series solution of the differential equation $(x^2-9)y''-2y'-4y=0$

a) Interval of converge

$x^2-9=0 \to \sqrt{x^2}=\pm\sqrt{9} \to x=\pm 3$

$\rho=|\pm 3-0|=3 \quad (0\pm 3, 0+3) \to \boxed{(\pm 3, 3)}$

finding the interval solving the $x^2-9=0$ and the absolute value of the result $\to (\pm 3, 3)$

b) $y=\sum_{n=0}^{\infty} C_n x^n \qquad y''=\sum_{n=0}^{\infty} C_n n(n-1)x^{n-2}$

$y'=\sum_{n=0}^{\infty} C_n n x^{n-1}$

Find the first and second derivate of the series

$LS=\sum_{n=0}^{\infty} C_n n(n-1)x^n - \sum_{n=2}^{\infty} 9C_n \cdot n(n-1)x^{n-2} - \sum_{n=1}^{\infty} 2C_n n x^{n-1}$

A B C

$-\sum_{n=0}^{\infty} 4C_n x^n$

D

Plug the series into the equation

Keep the x^n same in the equation

B $\sum_{n=2}^{\infty} 9n(n-1)C_n x^{n-2} \to \sum_{n=0}^{\infty} 9(n+2)(n+1)C_{n+2}x^n$

C $\sum_{n=1}^{\infty} 2nC_n x^{n-1} \to \sum_{n=0}^{\infty} 2(n+1)C_{n+1}x^n$

$\sum_{n=0}^{\infty} n(n-1)C_n x^n - \sum_{n=0}^{\infty} 9(n+2)(n+1)C_{n+2}x^n - \sum_{n=0}^{\infty} 2(n+1)C_{n+1}x^n - \sum_{n=0}^{\infty} 4C_n x^n$

$=\sum_{n=0}^{\infty} [n(n-1)C_n - 9(n+2)(n+1)C_{n+2} - 2(n+1)C_{n+1} - 4C_n]x^n$

$=\sum_{n=0}^{\infty} [-9(n+2)(n+1)C_n - 2(n+1)C_{n+1} + (n(n-1)-4)C_n]x^n$

$\boxed{=\sum_{n=0}^{\infty} [-9(n+2)(n+1)C_{n+2} - 2(n+1)C_{n+1} + (n^2-n-4)C_n]x^n}$

symplify the equation to keep the sum and x^n out of the equation.

c) $\boxed{C_{n+2}=\frac{2(n+1)C_{n+1}-(n^2-n-4)C_n}{-9(n+2)(n+1)}}$

Solve the equation by C_{n+2}

D) $C_0=? \quad C_1=? \quad C_2=-\frac{1}{9}C_1-\frac{2}{9}C_0$

Find the first terms by leaving C_0 and C_1 as variable

$n=1$

$C_3=\frac{2(1+1)C_2-(1^2-1-4)C_1}{-9(1+2)(1+1)}$

$\boxed{C_3=\frac{-16}{243}C_1+\frac{1}{243}C_0}$

$C_4=\frac{2(2+1)C_3-(2^2-2-4)C_2}{-9(2+2)(2+1)}$

$\boxed{C_4=\frac{50}{8748}C_1+\frac{1}{243}C_0}$

$y=C_0+C_1x+C_2x^2+C_3x^3+C_4x^4$

$=C_0+C_1x+(-\frac{1}{9}C_1-\frac{2}{9}C_0)x^2+(-\frac{16}{243}C_1+\frac{1}{243}C_0)x^3$
$+(\frac{50}{8748}C_1+\frac{1}{243}C_0)x^4$

enter the values into the first five terms of the series

$\boxed{y=C_0\left(1-\frac{2}{9}x^2+\frac{1}{243}x^3+\frac{1}{243}x^4\right)+C_1\left(x-\frac{1}{9}x^2-\frac{16}{243}x^3+\frac{50}{8748}x^4\right)}$

symplify and leave C_0 and C_1 as variables

e) $y(0)=-1 \quad , \quad y'(0)=2$

$=-1\left(1-\frac{2}{9}x^2+\frac{1}{243}x^3+\frac{1}{243}x^4\right)+2\left(x-\frac{1}{9}x^2-\frac{16}{243}x^3+\frac{50}{8748}x^4\right)$

replace -1 into C_0 and 2 into C_1 and simplify

$\boxed{=-1+2x-\frac{33}{243}x^3+\left(\frac{100}{8748}-\frac{1}{243}\right)x^4}$