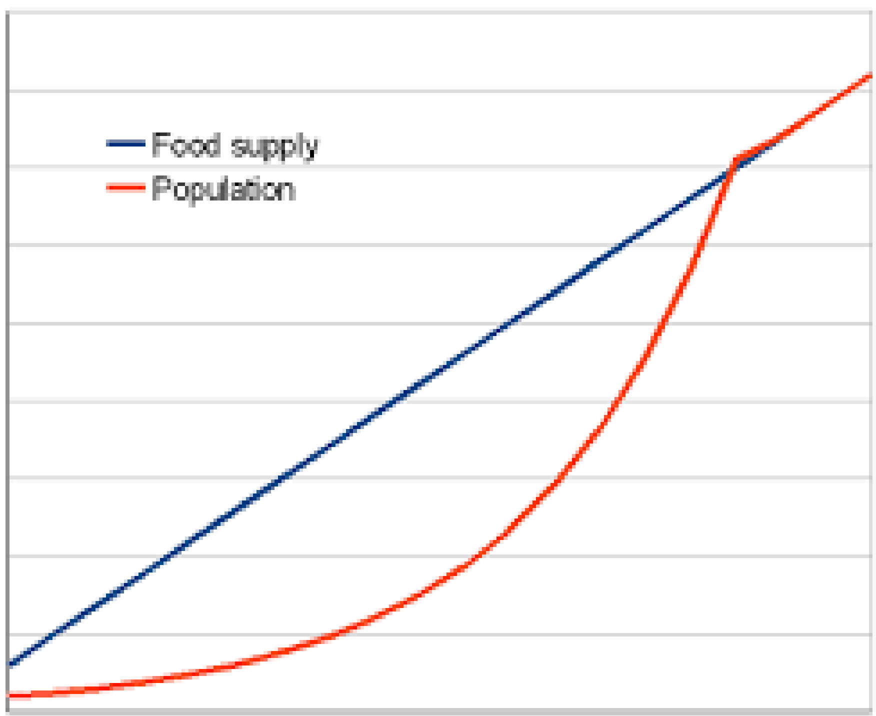
NEW YORK CITY COLLEGE OF TECHNOLOGY
CITY TECH
WHERE CAN TECHNOLOGY TAKE YOU?

Determining the Population Growth Effects On The Food Supply

Kevin Jaisingh
Differential Equation Mat 2680 D772
Project # 3

Everyday the population on Earth is increasing by hundreds of thousands of people.The population growth rate as of 2020 is around 1.05%. A decrease from 2019 where it was 1.08% and in 2018 it was 1.10%. We have around 7.9 billion people living on Earth right now. [1]This means the more people, the more houses, schools, jobs, restaurants and more will be needed as the population grows. All of these are important because the more people, the more space is needed to fit all of everyone. However there is one thing that is very important besides space and that is the food supply. The more people that there are, the more water and food is needed for everyone to survive. We need three meals per person everyday. That is a lot of food needed and that is something we cannot guess on. As the population grows the more food is needed because of how fast the population is growing. It's not so easy to make an assumption of how much food supply is needed to support the total population.



This graph shows the human population vs the food supply. As the human population increases, the food supply will have to increase. One cannot survive without the other. If there's not enough humans, a lot of food supply will go to waste. If there's not enough life stock, humans will starve. But this just shows the population and livestock at one time. We need to find out the population everyday to ensure that there is a lot of life stock. .

Before we can see how much food is needed. We need to know how big the population is right now. There are many ways to find the population growth and see how much increase in the food supply will be but the best way is with Differential Equations. In Differential Equations there are multiple ways to solve this type of question. There is Exponential growth that needs one population variable. This model is the rate of growth being equal to the separable size of the population. This model is incompleted. The equation is $dP/dt = kP$ with $P(0) = P_0$. Now, we would integrate the equation to get $\int dP/kP = \int dt \longrightarrow P(t) = Ae^{kt}$ In this case the A is derived from the constant of integration and can be found by using the initial condition. But there is another equation we will be using and that is the Verhulist model.[2] That is a logistic differential equation. This equation was published in 1845 by Pierre Verhulist. Verhulst had said the population growth will slow down when the population is large because of limited resources. The amount of resources available determines how large of a population there can be.

The equation is $dP/dt = rP(1 - P/K)$. [3] The "K" represents the carrying capacity for one set of population in the given environment. The "r" is a real number that represents the growth rate. P(t) represents the populations as a function at a time.P_0 is the initial population. This differential equation can be combined with the initial condition $P(0) = P_0$ to form an initial value problem P(t).

The steps to solve a problem with growth Population using the logistic differential equation.

1. First set the right hand side equal to zero leads to P = 0 and P = K as constant solutions. The first solution indicates the population will not grow because no organism will be represented. The second solution indicates the population starts at the carrying capacity, it will never change.
2. Now rewrite the differential equation in the right form.
   $dP/dt = rP(K - P)/K$
3. Multiply both sides by dt and then divide both sides by P(K - P).
   $dP/P(K - P) = (r/K)dt$
4. Now multiply K on both sides and integrate.
   $\int K/P(K - P)\, dP = \int rdt$
5. Use partial fraction decomposition on the left side of the equation.
   $K/P(K - P) = (1/P) + (1/K - P)$

   $\int 1/P + (1/K - P)dP = \int rdt$
   $\mathrm{Ln}|P| - \mathrm{Ln}|K - P| = rt + C$
   $\mathrm{Ln}|P/K - P| = rt + C$
6. Now eliminate the natural logarithms by using exponentiate on both sides.
   $e^{\mathrm{Ln}|P/K - P|} = e^{rt + c}$
   $|P/K - P| = e^{C} * e^{rt}$.
7. $C_1 = e^C$ So the equation will be
   $P/K - P = C_1 e^{rt}$.
8. To solve this equation, move all the Ps on the left side. Start off by multiplying K - P on both sides.
   $P = C_1 e^{rt}(K - P)$.
   $= C_1 Ke^{rt} - C_1 Pe^{rt}$
   $P + C_1 Pe^{rt} = C_1 Ke^{rt}$
9. Try and remove the factors that are not P on the left side.

$P(1+C_1 e^{rt}) = C_1 Ke^{rt}$

$P(t) = (C_1 Ke^{rt})/(1+C_1 e^{rt})$

10. Now solve for C_1.

$P/K - P = C_1 e^{rt})$

$P_0/ K - P_0 = C_1 e^{r(0)}$

$C_1 = P_0/ K - P_0$

11. Now substitute the expression C_1 into the equation.

$P(t) = (C_1 Ke^{rt})/(1+C_1 e^{rt}) = (P_0/ K - P_0)Ke^{rt} /1 + (P_0/ K - P_0) e^{rt}$

12. Now multiply the numerator and denominator of the right hand side by $K - P_0$ and then simplify.

$P(t) = (P_0/ K - P_0)Ke^{rt} /1 + (P_0/ K - P_0) e^{rt} * (K - P_0)/(K - P_0) = (P_0Ke^{rt})/(K - P_0) + P_0e^{rt}$.

Question:

Now that we understand how to use the Pierre Verhulist model, let's try and attempt to solve a problem using this equation. The problem is from [3] I did not write it.

Example 8.4.1: Examining the carrying capacity of a Deer Population.

Let's consider the population of white tailed deer (Odocoileus virginianus) in the state of Kentucky. The Kentucky Department of Fish and Wildlife resources (KDFWR) sets guidelines for hunting and fishing in the state. Before the hunting season of 2004, it estimated a population of 900,000 deer. Johnson notes:" A deer population that has plenty to eat and is not hunted by humans or other predators will double every three years" (Georege Johnson, " The Problem of Exploding Deer Populations Has No Attractive Solution," January 12, 2001. Accessee April 9, 2015). This observation corresponds to a rate of increase r = ln(2)/3 = 0.2311, so the approximate growth rate is 23.11% per year.(This assumes that the population grows exponentially, which is reasonable at least in the short term with plentiful food supply and no predators.) The KDFWR also reports deer population densities for 32 counties in Kentucky, the average of which is approximately 27 deer per square mile. Suppose this is the deer density for the whole state (37,732 square miles). The carrying capacity K is 39,732 square miles times 27 deer per square miles, or 1,072,764 deer.

a. For this application, we have P_o = 900,000, K = 1,072,764, and r = 0.2311. Form the initial value problem.
b. Solve the initial value problem from part a.
c. According to this model, what will be the population in 3 years? Recall that the doubling time predicted by Johnson for the deer population was 3 years.

Solution:

a. The initial value problem
   dp/dt = 0.2311P(1 - P/1,072,764) , P(0) = 900,000

b. Now we have this differential equation. Let's solve it. Since this is a differential equation, let's use the separation of variables methods to solve this initial value problem.

Step 1 - Let's set the right side equal to 0.

$P = 0$ and $P = 1{,}072{,}764$

This shows if the population starts at zero, it will never change, but if it starts at a capacity it will change.

Step 2 - Rewrite the differential equation. Multiple both sides.

$dp/dt = 0.2311P(1{,}072{,}764 - P/1{,}072{,}764)$

$dP/P(1{,}072{,}764 - P) = 0.2311/1{,}072{,}764\ dt$

Step 3- Use partial fractions decomposition to integrate both sides.

$\int dP/P(1{,}072{,}764 - P) = \int 0.2311/1{,}072{,}764\ dt$

$1/1{,}072{,}764 \int (1/P + 1/1{,}072{,}764 - P)\ dP = 0.2311t/1{,}072{,}764 + C$

$1/1{,}072{,}764\ (\ln|P| - \ln|1{,}072{,}764 - P|) = 0.2311t/1{,}072{,}764 + C$

Step 4 - Multiple both sides by 1,072,764. Use the question rule for this part.

$\mathrm{Ln}|\ P / 1{,}072{,}764 - P\ | = 0.2311t + C_1$

Step 5 - Get rid of the absolute value by using exponentiate on both sides. $C_1 = 1{,}072{,}764C$.

$e\ (\mathrm{Ln}|\ P / 1{,}072{,}764 - P\ |) = e(0.2311t + C_1)$

$|\ P / 1{,}072{,}764 - P\ | = C_2\ e0.2311t$

$P / 1{,}072{,}764 - P = C_2\ e0.2311t$

Step 6 - $C_2 = e^c{}_1$ Now solve.

$P = C_2\ e(0.2311t)(1{,}072{,}764 - P)$

$P = 1{,}072{,}764C_2\ e(0.2311t) - C_2\ Pe(0.2311t)$

$P + C_2\ Pe(0.2311t) = 1{,}072{,}764C_2\ e(0.2311t)$

$P(1 + C_2\ e(0.2311t) = 1{,}072{,}764C_2\ e(0.2311t)$

$P(t) = (1{,}072{,}764C_2\ e(0.2311t)) /\ C_2\ e(0.2311t)$

Step 6 - Solve for C_2. $P(0) = 900{,}000$

$P / 1{,}072{,}764 - P = C_2\ e0.2311t$

$900{,}000/1{,}072{,}764 - 900{,}000 = C_2\ e0.2311(0)$

$900{,}000/172{,}764 = C_2$.

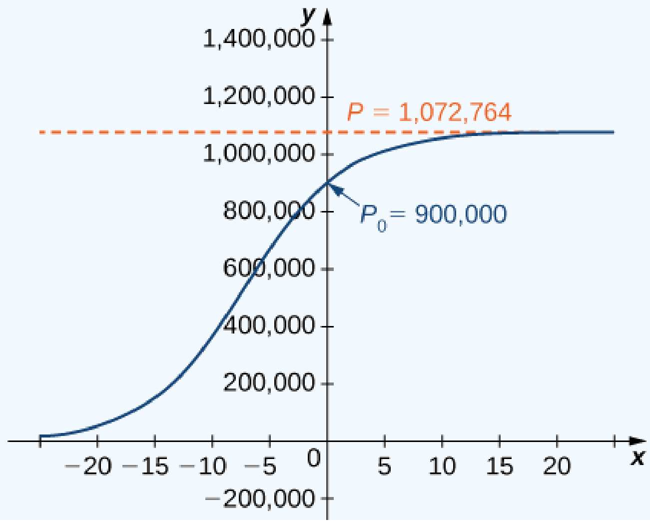
$5.209 = C_2$.

Therefore -

$P(t) = (1{,}072{,}764(5.209)e(0.2311t)) /\ (5.209)\ e(0.2311t)$

$P(t) = 1{,}072{,}764\ e^{0.02311t}/\ 0.19196 +\ e^{0.02311t}$

Figure is a graph of this equation.



This graph here shows the equation. It shows the carrying capacity of P = 1,072,764 and the P_0 = 900,000. This graph shows that the initial population of deers will be 900,000.

c. Now use the same model to predict the population in 3 years.

$$P(3) = 1{,}072{,}764e^{0.2311(3)} / 0.19196 + e^{0.2311(3)} = 978{,}830 \text{ deers.}$$

As you can see this equation allowed us to find out the population of deers in this environment. This tells us that there has to be enough food for 900,000 deers to survive in that environment. This equation is useful to know because it gives us a good number to make sure the food supply is at where it needs to be as the population grows.This example now gave us an idea about the deer population imagen how much success we would get if we used it for the human population. We would make sure our food supply is strong.

In conclusion,this is the Verhulst model and how to use it to solve a growth population by food supply question. We talked about how there are many different types of differential equations that could be used to solve problems like these, but we focused on the Verhulst model. The equation was published in 1845. He said the population was growing and was getting too large. This equation is what allowed him to see if our environment could handle the new increase of the population. This equation could be used to predict the population in years or the population of now. Knowing this number will help us know that our environment has a strong food supply to fit that population. We saw how to use this equation in differential equations.We saw step by step how to solve a question using this model. Then used it to answer a question about deers population. Doing this gave us the idea how the equation works and how successful it can be using this equation. Now that we have this we can track our population growth and see if the food supply will be able to handle that population size.

Reference -

[1].https://www.worldometers.info/world-population/#:~:text=Population%20in%20the%20world%20is%20currently%20%282020%29%20growing,late%201960s%2C%20when%20it%20was%20at%20around%202%25.

[2]https://www.projectrhea.org/rhea/index.php/Malthusian_Doctrine:_Population_Growth#:~:text=Verhulst%20claimed%20that%20as%20population,population%20the%20environment%20can%20support.

[3]https://math.libretexts.org/Courses/Monroe_Community_College/MTH_211_Calculus_II/Chapter_8%3A_Introduction_to_Differential_Equations/8.4%3A_The_Logistic_Equation#:~:text=Definition%3A%20Logistic%20Differential%20Equation,-Let%20K%20represent&text=dPdt%3DrP(1%E2%88%92PK,problem%20for%20P(t).