Project 3: Bungee Jumping

Jocelyn Nacimba

MAT 2680-D772

5/22/2023

Have you ever attempted something remarkable in your life? In today's world, extreme sports have become a popular activity as more and more people seek excitement in their lives. One extreme sport that is considered the most famous and thrilling is bungee jumping. Bungee jumping is an exhilarating adventure sport in which participants jump from high platforms such as bridges, cranes, or towers while being attached to a thick and specialized cord that retracts after a few seconds of freefall. Throughout the years there have been bungee jumping accidents, but to keep jumpers as safe as possible, numerous safety standards and guidelines have been established. It is necessary to make sure that the individual does not experience excessive acceleration and doesn't hit the ground or the bridge they jump off of. With the application of second order differential equations, the motion of bungee jumping can be modeled. The importance of analyzing this motion is to be able to know how to prevent bungee jumping catastrophes and to ensure the wellbeing of the person.

The motion with respect to time of bungee jumping can be described into two different sections: the free fall, and the time when the bungee cord is pulling on the jumper. Both sections can be modeled with Newton's second law of motion which states that the total of all the forces acting on an object is equal to mass of the object multiplied by its acceleration and is represented by the formula $\Sigma F = ma$. When the jumper makes their first jump, they experience free fall. Free fall is essentially when only the mass of the jumper is falling due to the force of gravity and wind resistance (F_W). The equation $F_W = \frac{mg}{v^2}$ establishes a constant relationship between jumper action and wind forces and is utilized to determine F_W. As previously mentioned, the sum of the forces is equal to the product of mass time acceleration ($\Sigma F = ma$), where ma is equal to the force due gravity plus the wind resistance, thus it can be written as $ma = mg + F_W$.

Hook's law is the force needed to stretch an elastic object such as a spring or in this case a bungee cord is directly proportional to that change in length. $F = -kx$, where k is the spring constant that indicates the stiffness of the bungee cord and x is the distance that the cord is stretched describes Hooke's law. It is important to note that the higher the k value, the harder it is to stretch the bungee cord.

After the jumper passes the free fall stage; the jumper, being at the end of the bungee cord, will exert a downward gravitational force on cord and this downward force is balanced by the restoring force of the bungee cord, which is the upward force acting on the cord, in equilibrium.

This will result in the bungee jumper going up and down. In other words, the bungee cord will act as a spring; expanding and compressing allowing the jumper to bounce up and down. They will still have gravity and wind resistance acting on them, but now they will also have the force of the cord pulling on them. Due to the fact that the cord is held from a vertical position with a mass at the end acting also vertically, this can be thought of as an inverted spring-mass with damping. With m serving as the mass, λ as the damping constant, and k as the spring constant, the differential equation that describes the motion of the bungee cord is $m\frac{d^2y}{dt^2} + \lambda\frac{dy}{dt} + ky = 0$ or $my'' + \lambda y' + ky = 0$. After a few seconds, the speed and height will decrease, and the jumper will stop moving; this is due to damping. There are different stages of damping: overdamped, critically damped, and underdamped. In both the overdamped and critically damped stage, the spring does not oscillate properly. While in the underdamped stage, it allows long undulations of the spring and soft returns. The damping constant is a constant introduced into an oscillatory spring equation to account for the reduction of motion as time progresses. If $\lambda^2 > 4mk$, then it is the overdamped stage, if $\lambda^2 = 4mk$, then it is the critically damped stage, and if $\lambda^2 < 4mk$, then it is the underdamped stage. Bungee jumping is connected with the underdamped stage.

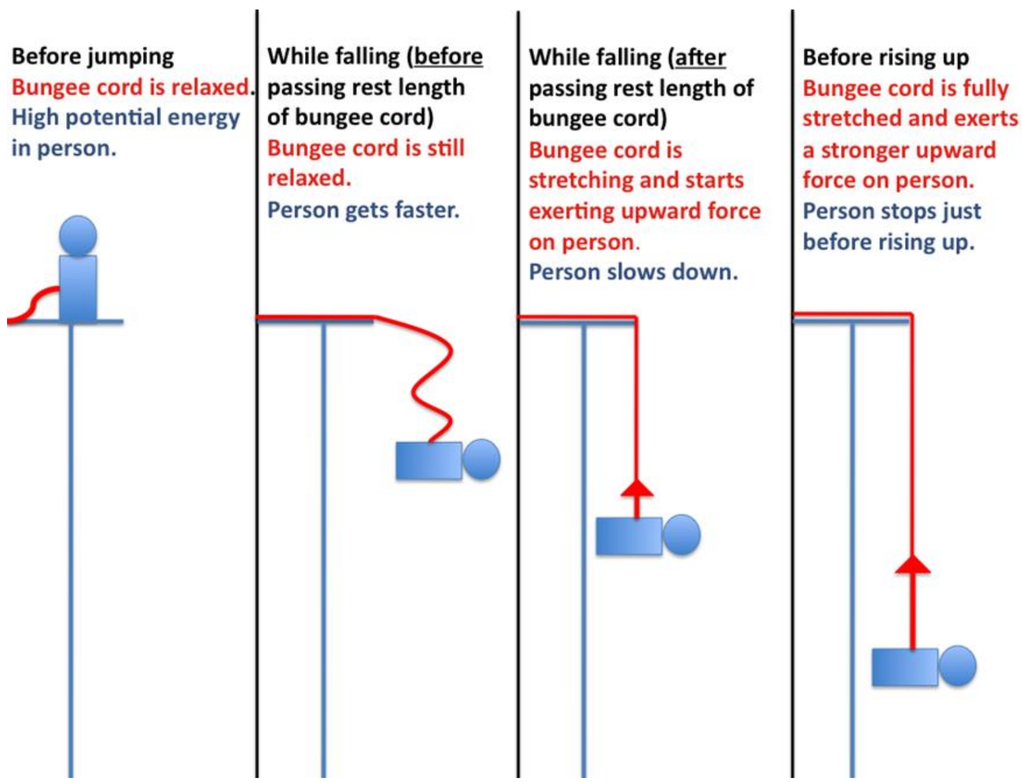

Figure 1: Bungee Jumping Motion

In conclusion, bungee jumping is an activity that everyone can enjoy, whether is to get that adrenaline rush and feel truly alive or to conquer a fear or even just to live a new adventure. However, often time what comes to mind is how safe bungee jumping actually is? There are many accidents that can occur in a blink of an eye and for most people it is considered a risk to go bungee jumping. Through the application of second order differential equations, we saw how bungee jumping actually works and how these equations play a significant role in making bungee jumping safe. During the free fall stage, the jumper falls due their weight and the force of gravity pulling them down. When designing a bungee cord, knowing the spring constant is important because it allows designers to know how much the cord has to stretch. Based on height and weight of a person, the bungee cord might have a higher spring constant or a lower spring constant in order ensure that the cord is able to have a sustainable hold of the person when free falling and to prevent the cord from breaking which can lead to the person falling. Passing the free fall stage, the jumper will bounce up and down because the bungee cord will act as a spring, where there will be the force of gravity pulling down and the force of the cord pulling in the opposite direction. In order for the jumper to not hit something as they are bouncing up and down, the spring constant is needed because it tells you how much the cord should expand and compress so it does not put the jumper in danger. Damping is another key component of bungee jumping. It allows the jumper to experience the trilling motion and to come to a safe stop. During the overdamped and critically damped, the jumper might experience too much acceleration which can put the jumper at risk if they have any medical condition. Aside from the jumper not being able to come to a safe stop, they might get a spine, neck, or eye injury. The damping constant determines whether the cord is set to an underdamped, overdamped, or critically underdamped case. Placing the proper parameter into the equation can illustrate the motion of the jump which can help depict any casualties and ensure that the jumper has a successful jump.

Example:

It is Jacob's birthday and his parents decided to take him bungee jumping which is an extreme sport he always dreamed of doing. Jacob, who weighs 70 kg stood on top of a bridge, attached to an underdamped bungee cord that is 50 m long. The bungee cord he's hooked up to has a spring constant of 1000 N/m and a damping constant of 3740 1/s. Find y(t).

First we need to find the equivalent weight of Jacob when he reaches the end of the rope. In other words we need to see how fast he's travelling when he reaches the 50 meter length of the rope.

$$t^2 = 2\frac{50}{9.8}$$

$$t = 3.2 \text{ s}$$

$$v = 9.8(3.2)$$

$$v = 31.36\ \frac{m}{s}$$

$$F = \frac{1}{2}(70)(31.36)^2$$

$$F = 34420.7$$

$$F_W = (70)\ \frac{9.8}{(31.36)^2}$$

$$F_W = 0.7 \text{ N}$$

$$\text{TotalF} = 34420 \text{ N}$$

$$\text{Weight} = 3512 \text{ kg}$$

Next, since we know our constants, we can set up the differential equation to solve the spring portion beyond the unstretched length of the cord

$$3512y'' + 3740y' + 1000y = 0$$

With this being our equation, we can conclude that 3512 represents the mass, 3740 represents the damping constant, and 1000 represents the spring constant. With those values known, we can then plug them into the formula for the general solution obtained from the derivation. The solution can be seen as

$$y(t) = Ce^{-(6567440t)}\cos(-12987600t)$$

References:

1) https://mse.redwoods.edu/darnold/math55/DEproj/sp14/EricMoon/FinalDraft_paper.pdf
2) https://www.youtube.com/watch?v=RL-FGItleqE