Differential Equations in Machine Learning

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Differential Equations

ORDINARY DIFFERENTIAL EQUATIONS

“ODE is an equation that contains only one independent variable and one or more of its derivatives with respect to the variable.”

Example: $y'' + 10y' + 9y = 0$

PARTIAL DIFFERENTIAL EQUATIONS

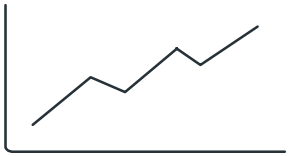
“An equation involving only partial derivatives of one or more functions of two or more independent”

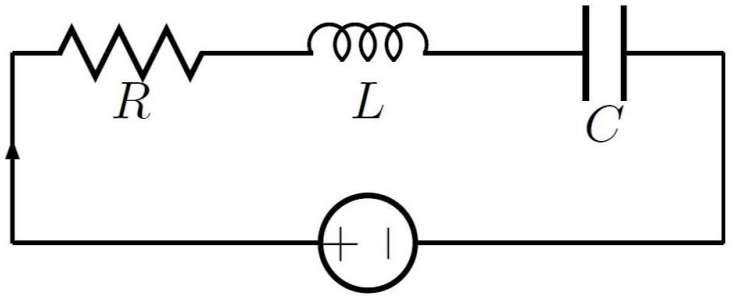
Example:

Heat Conduction Equation: $\frac{\partial T}{\partial t} = C\frac{\partial^2 T}{\partial x^2}$

Purpose of Differential Equations

- The objective of differential equations is to predict behavior of data in real life problems.





RLC Circuit Application

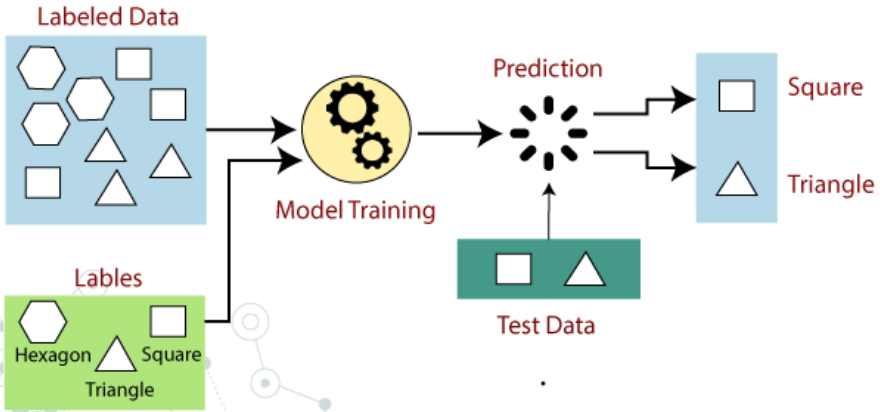
Machine Learning

- Machine learning can predict human actions.
- Solve complicated problems involving large data.
- It has two types of methods.

Types of methods

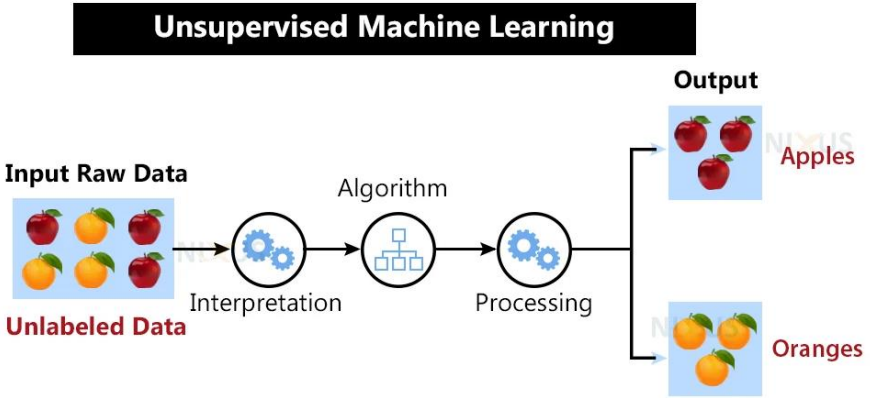
Supervised

- It uses both input and output data.
- Feedback as input.

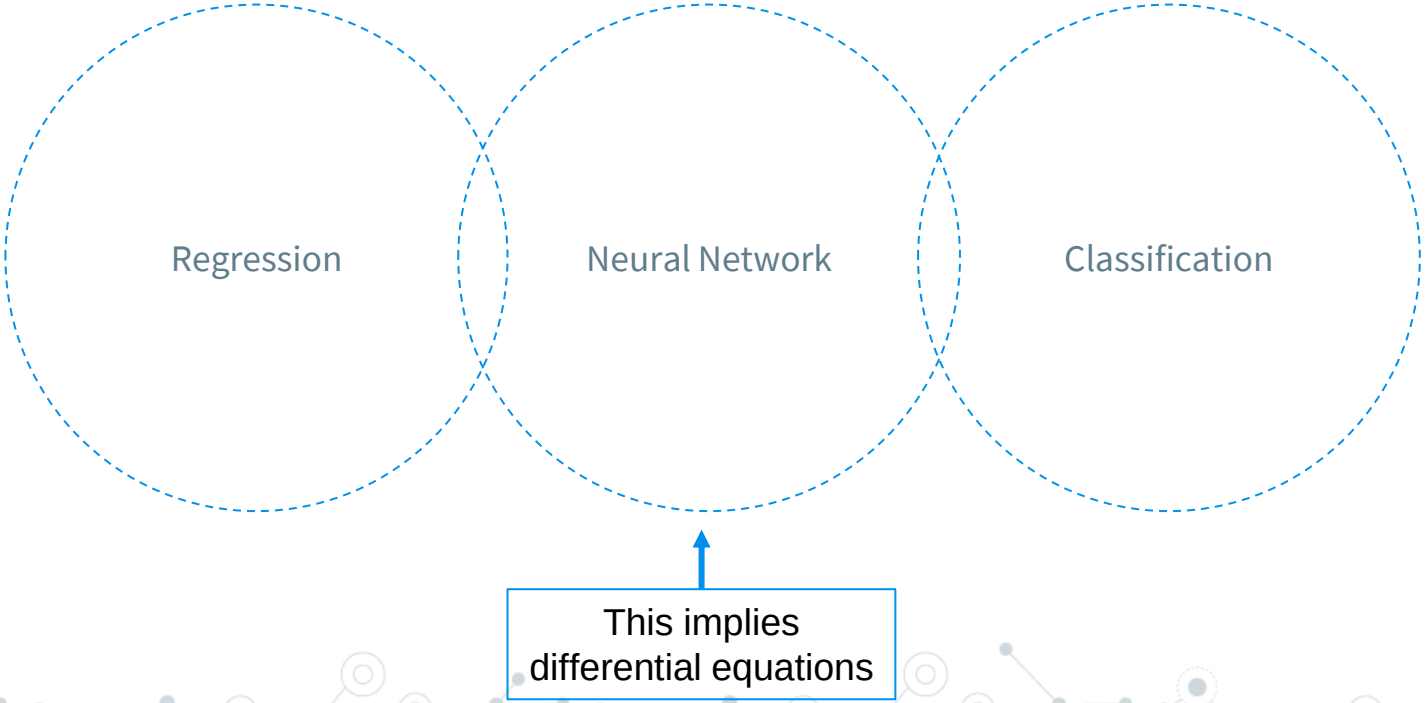


Unsupervised

- Only uses input.
- Interpretation.



Techniques of Machine Learning in supervised method



Neural Network Technique

According to Patrick, an oxford mathematician says that the purpose of this technique is to combine neural network which are focused on image recognition and understanding of language processing, with differential equations.

$$z(0) = z_0, \qquad \frac{\mathrm{d}z}{\mathrm{d}t}(t) = f_\theta(t, z(t))$$

Neural controlled by differential equation

In order to add a value of time into the equation, a new variable should be added to d(t).

For example, this variable can be used to compare the rate of change dz(t).

$$\mathrm{d}z(t) = f_\theta(t, z(t))\,\mathrm{d}X(t)$$

Example of using Neural Controlled DE

- Predicting which letter between a, b, and c a person will write with a pen.

Test accuracy (mean ± std, computed across five runs) and memory usage on CharacterTrajectories. Memory usage is independent of repeats and of amount of data dropped.

Model	Test Accuracy			Memory usage (MB)
	30% dropped	50% dropped	70% dropped	
GRU-ODE	89.9% ± 8.4%	89.6% ± 5.6%	86.6% ± 3.5%	1.5
GRU-Δt	94.4% ± 1.7%	92.0% ± 1.0%	91.1% ± 1.1%	15.6
GRU-D	93.2% ± 2.0%	92.7% ± 2.8%	90.8% ± 2.1%	16.9
ODE-RNN	97.9% ± 0.4%	97.5% ± 0.6%	96.7% ± 0.9%	14.7
Neural CDE (ours)	**99.2% ± 0.3%**	**99.3% ± 0.3%**	**99.4% ± 0.4%**	**1.3**

- Tests which words between “left” and "right” a person will say

Test Accuracy (mean ± std, computed across five runs) and memory usage on Speech Commands. Memory usage is independent of repeats.

Model	Test Accuracy	Memory usage (GB)
GRU-ODE	47.9% ± 2.9%	**0.164**
GRU-Δt	43.3% ± 33.9%	1.54
GRU-D	32.4% ± 34.8%	1.64
ODE-RNN	65.9% ± 35.6%	1.40
Neural CDE (ours)	**89.8% ± 2.5%**	0.167

IA Solving Differential Equations

- Since AI is experimented with many uses, an article from Medium intitled “Artificial Intelligence Can Now Solve Partial Differential Equations” state that:
- “Researchers at Caltech have introduced a new deep-learning technique named “Fourier Neural Operator for Parametric Partial Differential Equations” for solving PDEs that is dramatically more accurate than Deep learning methods developed previously.”

Thanks!

Cited work

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