How can the Differential Equations be important in Forecasting Hurricanes

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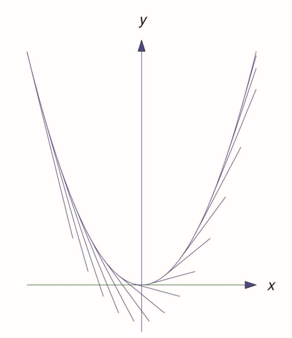
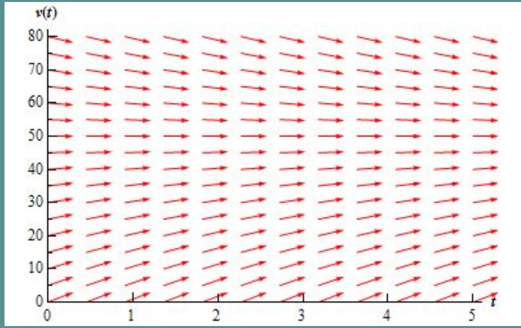
Know your weather!

- When people watch important weather news on television or radio, the meteorologist tells specific information about the direction that the hurricane hits and the hurricane's strength.

- The way they could determined in Forecasting Hurricane would be the use of Differential Equation. The first order equation as applications to curves, and the Direction Fields as dynamic models. These are mechanisms that would help people to receive accurate details for their latest forecast on hurricanes for years to come

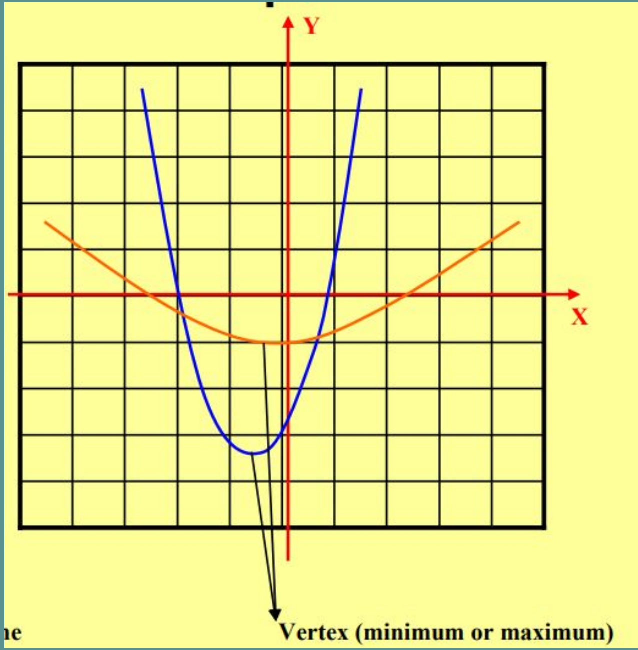




One example know as the Application of Curve (Part 1)

When a hurricane is spotted, people are determine to identify what course of direction it would take. As there are many possibilities from a city town, unpopulated areas, or a vast ocean. To do that they would have to use the curves, according to the MDC in side 36, they track a hurricane by using a parabola or parabolic: Y= ax2+ bx + c

As stated, the graphs depict where if a > 0 the direction will curve upwards. But if a < 0, then the direction will curve downwards.



One example know as the Application of Curve (Part 2)

- If we compare it to the Digital Commons textbook, it follows the same logic. On pages 179 and 180, a given example was shown: y - cx^2 = 0 (Figure 4.5.1)
- This equation follows the curve in the graph and various directions of the curve in upwards and downward which depending amount of parameter of c. This is similar to the ‘a’ from MDC. In other words, both the x and y function as cause and effect principles. Where x acts as the core of influencing the parameter and y acts are the impact given by the parameter. As such, the x,y, and any parameters like ‘a’ and ‘c’ are defined as a *one-parameter family of curves.*

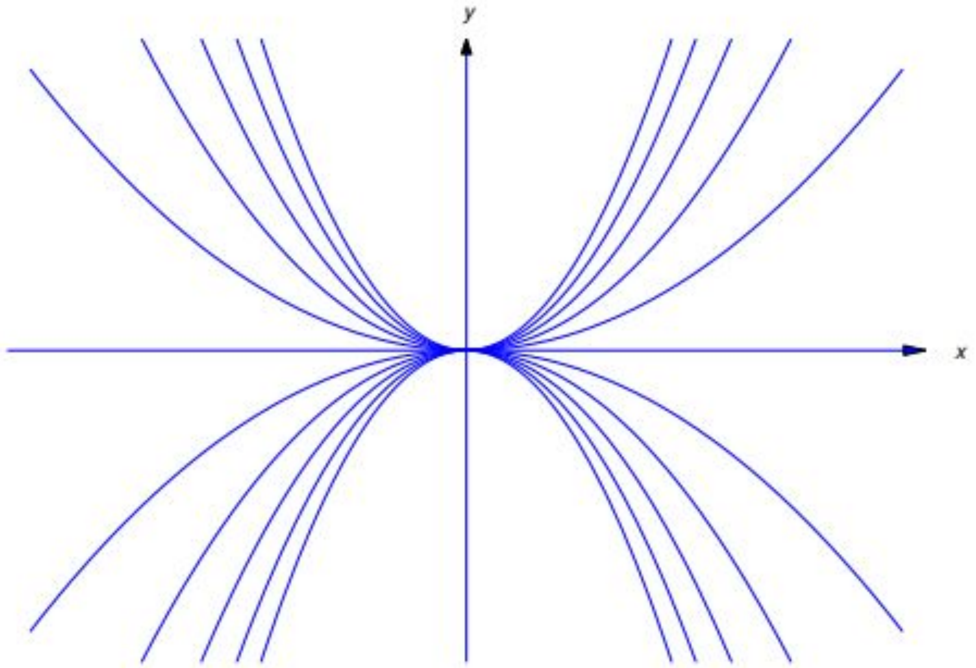


Figure 4.5.1 A family of curves defined by $y - cx^2 = 0$

Another example of Application of Curve (Part 1)

Going even further, we will look at parabola/parabolic. We understand that individuals who work on weather prediction use “Time-Stepping Schemes” According to Numerical Weather Prediction, their goal is to reach the high expectation of reaching their exact forecast range. They include a 1st order of equations known as the Linear differential equation which is used to apply the implicit schemes. They first start by forming Q to indicate a typical dependent variable direct from an equation that forms into dQ/dt=F(Q). Sinces, it is derivative of the quantity ‘Q’ with respect to quantity ‘t′. With it, they can replace the continuous-time domain t in a sequence of discrete times :[0, t, 2t, …nt,..]. Including, a solution directly indicate by Q^n = Q(nt). In addition, if the requirement of the solution is noted up to time: t = nt, then the right-hand term F^n = F(Q^n) would be calculated. This would make the time(t) with derivate into an approximated by centered differences:

$$\frac{Q^{n+1} - Q^{n-1}}{2\Delta t} = F^{n}$$

This centered difference of the“forecast” value Q^n+1 possibly calculated from the previous/old value Q^n-1 and the tendency Fn:

$$Q^{n+1} = Q^{n+1} + 2\Delta t F^{n}$$

Another example of Application of Curve (Part 2)

This new form is called leapfrog scheme. It is capable to learn that stepping forward from moment to moment is repeated a large number of times until their goal is to reach the high expectation of reaching their exact forecast range. The leapfrog scheme has some limitations by principle stability that limits the size of the time step Δt. There is an idea that could get around the limitation, using the implicit scheme that I mention before known as. The formation of an implicit scheme would be: →

$$\frac{Q^{n+1} - Q^{n-1}}{2\Delta t} = \frac{F^{n-1} + F^{n+1}}{2}$$

* This would allow continuing time step without any restriction.

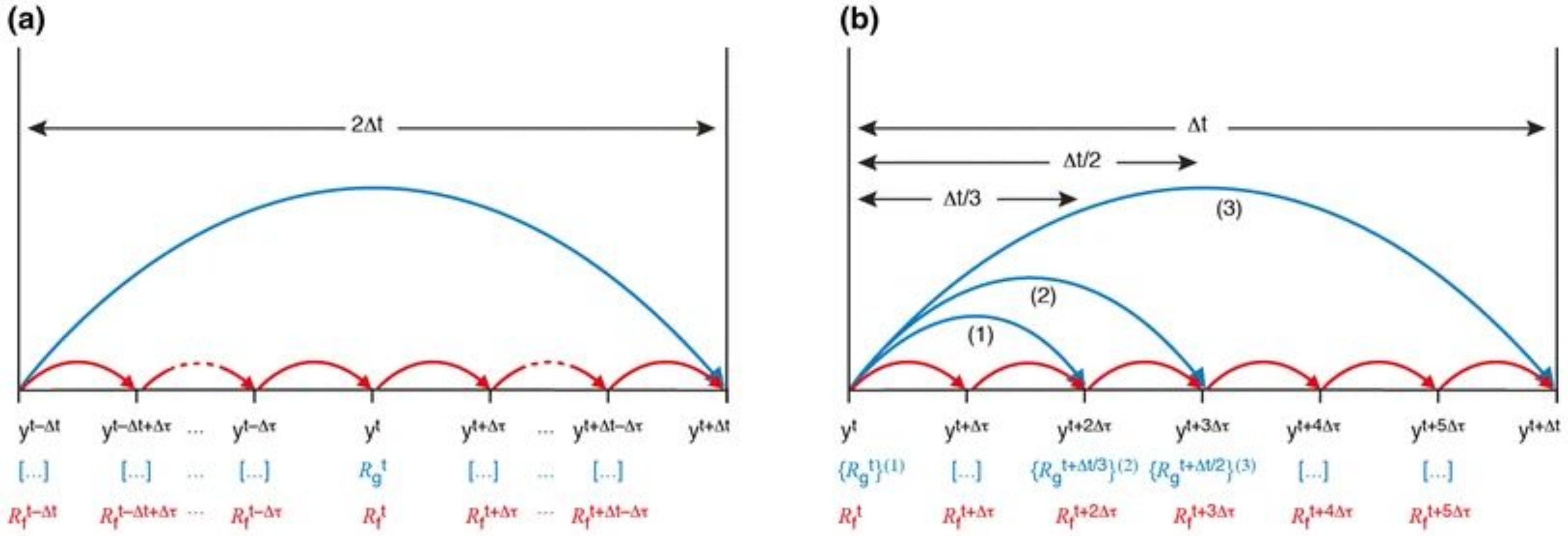
However this scheme would require an additional solution of an equation to make the process possible: →

$$Q^{n+1} - \Delta t F^{n+1} = Q^{n-1} + \Delta t F^{n-1}$$

- It mentions that this is considered forbidding, fortunately, it possible by having F(Q) represents as a Linear function. In other words, this implicit scheme can be operated only on particular (linear) meaning the 1st order of equations known as the Linear differential equation.

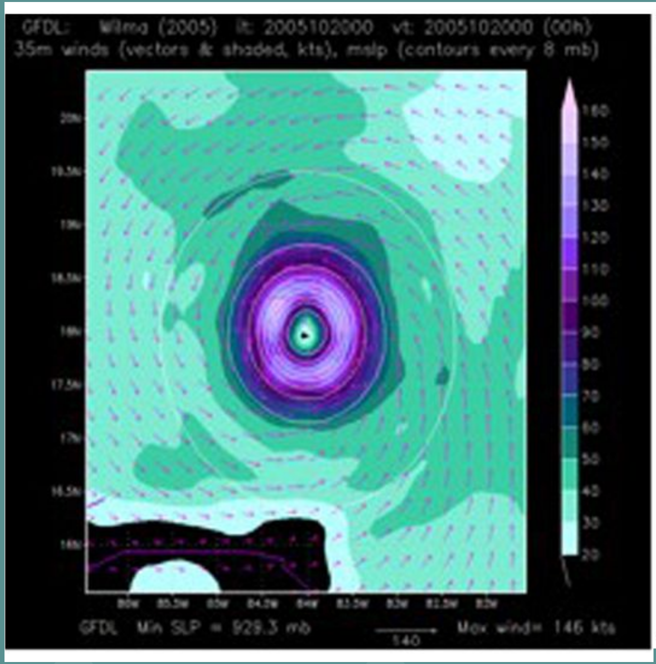
An illustration of Application of Curve with parabola/parabolic

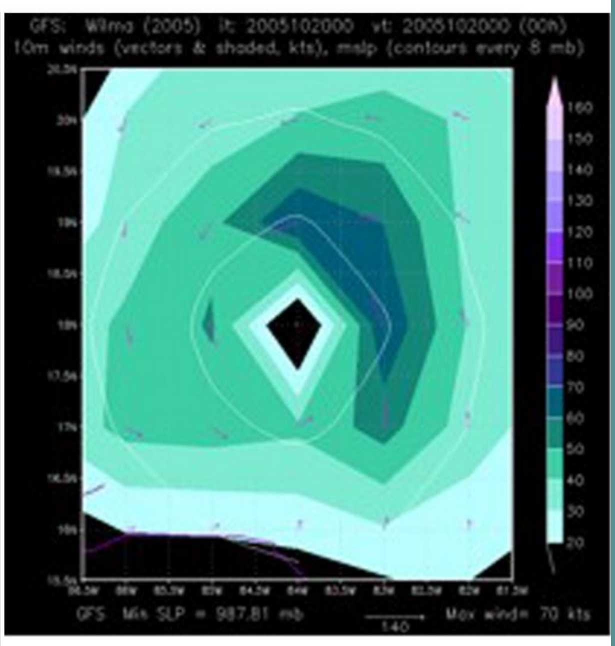
“Time-Stepping Schemes” and “leapfrog scheme”



Direction fields as dynamic models explanation! (Part 1)

Another method of determining the direction of hurricanes was the use of direction fields as dynamic models. According to Hurricane Science, they do this by a system called horizontal boundary condition, which would record accurate track and intensity for its forecasts. In their model, there is a wind direction as arrows with other backgrounds such as wind speed with a set that resembles a graph where the x and the ‘y’ are data of distances. The wind direction allows them to depict the hurricane's movement and understand the strength that the hurricane is providing. This is considered to be the most plausible way people use for the weather of hurricanes and parabole/parabolic.





Direction fields as dynamic models explanation! (Part 2)

Looking back to the Digital Commons textbook, on pages 16 to 17. They also face a similar difficulty and create a similar measure to find their solution. Since it is complicated to find complete solutions for various differential equations, they have to use graphical methods to know how equations perform. To do that they use an existing first-ordered equation solution y`= f(x, y), then recall that this is also a function of y = y(x). Plugging in would be y`(x) = f(x, y(x)). With this, it can be calculated with slopes of curves known as the slope of an integral curve through a given point (Xo, Yo) provided in f(Xo, Yo). This would be the start of direction fields. Now, if the following f can be an R set, it can construct a direction field for y`= f(x, y) by having multiple tiny segments like arrows on each (x, y) point on R with slope f(x, y). But, since it we cannot waste any more time plotting every point in R, we can create a set of finite points in R. Among which having f become a closed rectangular region.

$$R: \{a \le x \le b, c \le y \le d\}$$

Have point [a,b,c,d] be the equally spaced point: $a = x0 < x1 < \dots < xm = b$
and $C = y0 < y1 < \dots < yn = d$. With these, it formed points, we would assume to be (xi,yi), $0 \le I \le m$, $0 \le j \le n$ would become the *rectangular grid*(located in figure 1.3.1) where it would place the finite amount of short line segment with possible slope, this can allow us to create an integral curve. In addition, a numerical method will allow us to plot solution curves in the rectangular grid if f is continuous on R, then many formations will form based on figure 1.3.2 to figure 1.3.4. These formations look similar and possibly an inspiration for the idea of horizontal boundary conditions as both follow plot solution curves that provide directions and are even continuous in their graph form.

Direction fields as dynamic models illustration!

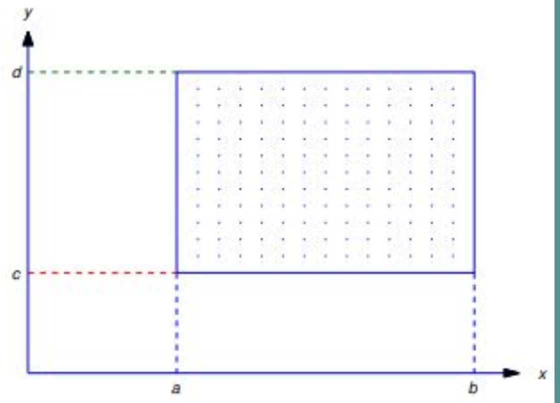


Figure 1.3.1 A rectangular grid

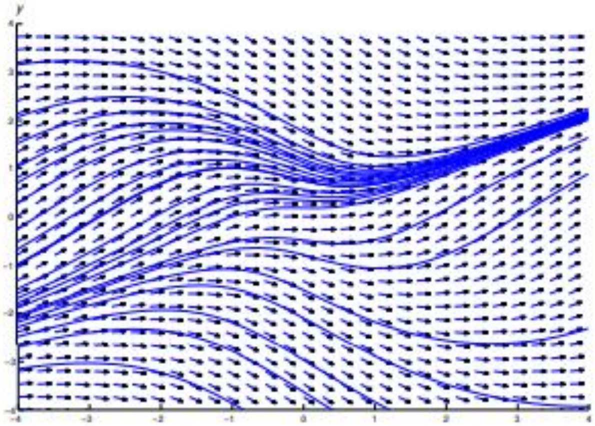


Figure 1.3.2 A direction field and integral curves for $y = \dfrac{x^2 - y^2}{1 + x^2 + y^2}$

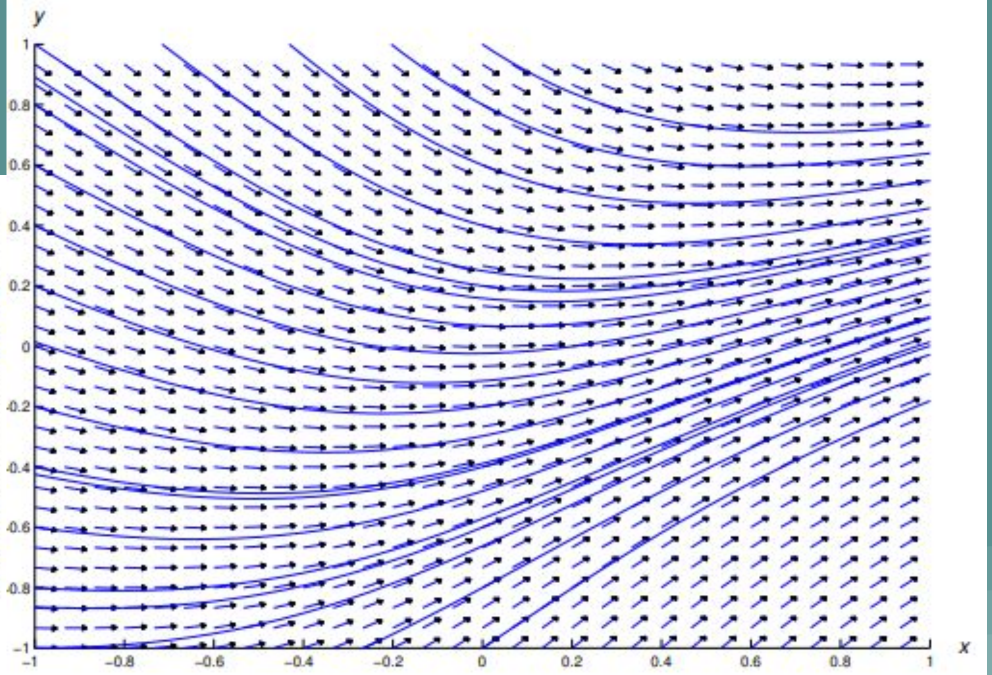


Figure 1.3.4 A direction and integral curves for $y' = \dfrac{x - y}{1 + x^2}$

Direction fields as dynamic models explanation! (Part 2)

Going even further on the differential field, we understand that individuals who work on weather prediction use the "Spatial Method." According to Numerical Weather Prediction, they first want to find the advanced coefficient of time after new information about the physical fields can be computed. They include an ordinary differential equation of the spectral coefficients: → Q_n^m

After the model equation is changed into a spectral space that becomes many equation sets. They do this by having an expanded field of spherical harmonics: →

$$Q(\lambda, \phi, t) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} Q_n^m(t) Y_n^m(\lambda, \phi),$$

Direction fields as dynamic models explanation! (Part 3)

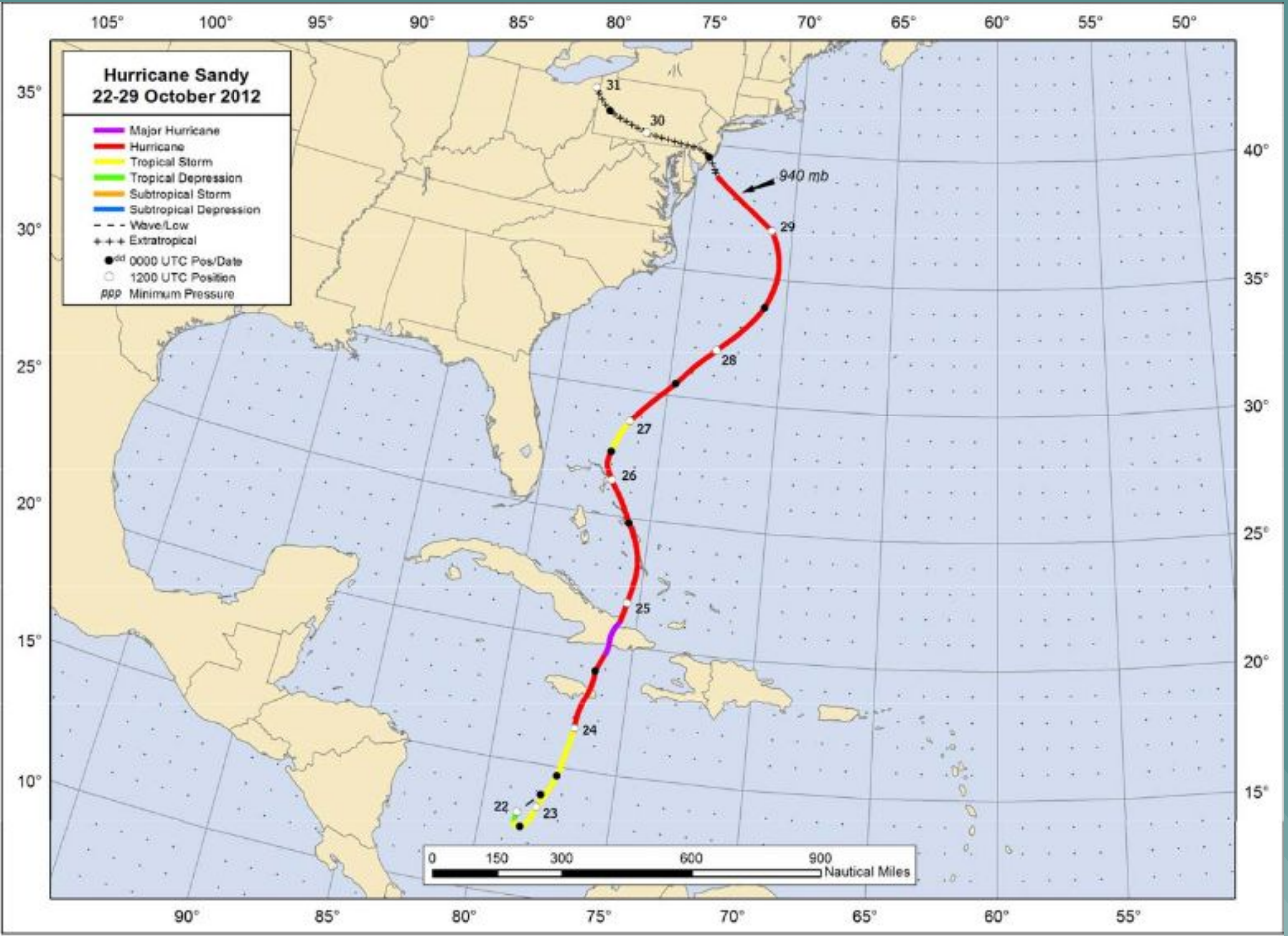
Each set function had a different purpose such as Q_n^m works on time, $Y_n^m(\lambda, \Phi)$ being as spherical harmonics, which is equal to $\exp(im\lambda)P_n^m(\Phi)$ which is used for longitude λ and latitude Φ. These are the ones that will be then transformed into ordinary differential equations for the spectral coefficients Q_n^m.

As how they tried to solve, they first need to have the expanded field of spherical harmonics become truncated at once:

$$Q(\lambda_i, \phi_j, t) = \sum_{n=0}^{N} \sum_{m=-n}^{n} Q_n^m(t) Y_n^m(\lambda_i, \phi_j).$$

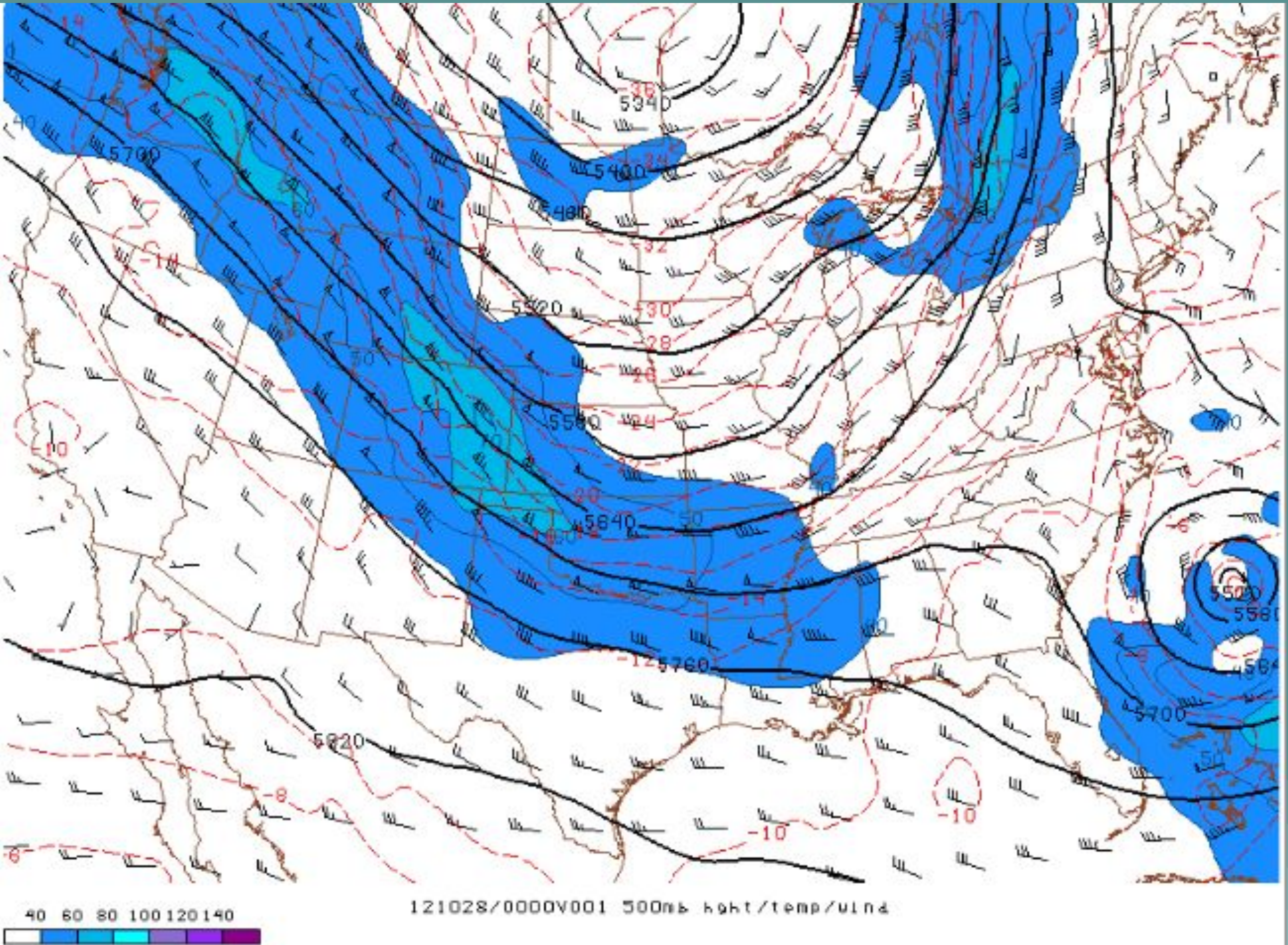
This becomes a triangular truncation with the value of N that can specify a simulation of model quality, As they have a computational grid named the Gaussian Grid that can function with the spectral truncation. This is considered another most plausible way people use for weather like hurricanes and differential fields.

Real World Event and why we would use differential equation for incoming Hurricane (Part 1)



The reason why we have these information is because, a natural disaster occur year 2012, which was Hurricane Sandy. According to the US Department of Commerce in both the Meteorology section and the Video, hurricane Sandy went from the Caribbean and moved north by the warm Gulf Stream until it went on a sharp turn to the area between New Jersey and New York and collided with a winter-like storm system that results in Hurricane Sandy to become huge and strong that lead massive devastation when it reaches land. According to the data given by the US Department of Commerce in the Meteorology section under the NHC sandy report, that sharp turn into a wide parabola. In theory that the parameter must have a lower value, as stated by the MDC, the "Large the value of a narrower a parabola look like".

Real World Event and why we would use differential equation for incoming Hurricane (Part 2)



In addition, based on the 500mb and the Surface data from the US Department of Commerce in the Meteorology section, both of them seemed to use a familiar direction field but with time elapsed. The 500mb has arrows that are represented as various wind speed symbols. While the surface has arrows that depict what direction is the window going. Including the tracing curve that crosses through the wind speed symbols and arrows. From there when the hurricane strike, the wind symbol shows the wind speed symbol changes into a symbol indicating that the wind has gotten stronger, and the arrow shifts from a curve into loops, where the tracing even change from a curve into a circle.

Final Thoughts

In conclusion, using the aspect of differential equations would be able to help individuals to analyze the possibilities for forecasting hurricanes such as the direction where the hurricane is going and how strong the hurricane becomes. A differential equation is considered to be tedious and complicated at first. But this example makes the analyses much more understandable when you want to know about the hurricane's current activity. As such, I hope we can use the differential equation for future forecasts.

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