Problem 9 (a)

Our interval for $(x-5)y'' - xy' - 2y = 0$

$\frac{2}{2}$ centred at 0

$a_0(x) = x-5$

$0 = x-5$

$\therefore x = 5$

$\therefore \rho = (-5, 5)$

Problem 9 (b)

$\frac{5}{5}$ $(x-5)y'' - xy' - 2y = 0 \quad \ldots (i)$

$\therefore$ Our $y = \sum_{k=0}^{\infty} C_k x^k \quad \ldots (ii)$

$y' = \sum_{k=0}^{\infty} k C_k x^{(k-1)} \quad \ldots (iii)$

$y'' = \sum_{k=0}^{\infty} k(k-1) C_k x^{(k-2)} \quad \ldots (iv)$

So we subs eqn (ii), (iii) and (iv) into eqn (i)

$$(x-5)\sum_{k=0}^{\infty} k(k-1)C_k x^{(k-2)} - x\sum_{k=0}^{\infty} kC_k x^{(k-1)} - 2\sum_{k=0}^{\infty} C_k x^k$$

we distribute into the summations

$$\underbrace{\sum_{k=0}^{\infty} k(k-1)C_k x^{(k-1)}}_{a} - \underbrace{\sum_{k=0}^{\infty} 5k(k-1)C_k x^{(k-2)}}_{b} - \underbrace{\sum_{k=0}^{\infty} kC_k x^k}_{c} - 2$$

Since our c and d has only x^k then we don't need to shift them. a and b needs shifting so we can have x^k on all part.

(a) $\displaystyle\sum_{k=0}^{\infty} k(k-1)C_k X^{(k-1)} \underset{\text{Peel}}{=} \underbrace{(0)(0-1)C_0 X^{(0-1)}}_{0} + \sum_{k=1}^{\infty} k(k-1)C_k X^{(k-1)}$ ✓ (at $k=0$)

$\displaystyle \underset{\text{Shift } 1}{=} \sum_{k=0}^{\infty} (k+1)(k)C_{k+1} X^{k}$ ✓. The reason why we are peeling using $k=0$ is because we are shifting by 1. ✓

(b) $\displaystyle\sum_{k=0}^{\infty} 5k(k-1)C_k X^{(k-2)} \underset{\text{Peel}}{=} \underbrace{5(0)(0-1)C_0 X^{(0-2)}}_{0} + \underbrace{5(1)(0-1)C_1 X^{(1-2)}}_{0} +$ (at $k=0$) (at $k=1$)

$\displaystyle \sum_{k=2}^{\infty} 5k(k-1)C_k X^{(k-2)}$ ✓ $\displaystyle\underset{\text{Shift } 2}{=} \sum_{k=0} 5(k+2)(k+1)C_{k+2} X^{k}$ ✓

in this case we are shifting by 2 which is why we peeled with $k=0$ and $k=1$.

Now we bring them together to collect like terms. ✓

$$\sum_{k=0}^{\infty}(k+1)(k)C_{k+1}X^k - \sum_{k=0}^{\infty} 5(k+2)(k+1)C_{k+2}X^k - \sum_{k=0} kC_kX^k - 2\sum_{k=0} C_kX^k$$

from here we factor out $\sum_{k=0}^{\infty}$ and X^k then we try to further simplify it. OK

$$\sum_{k=0}^{\infty}\left[(k+1)(k)C_{k+1} - 5(k+2)(k+1)C_{k+2} - kC_k - 2C_k\right]X^k$$ ✓

great!

re-writing:

$$P: \sum_{k=0}^{\infty}\left[(k+1)(k)C_{k+1}-5(k+2)(k+1)C_{k+2}-kC_k-2C_k\right]x^k$$

collecting like terms

$$\sum_{k=0}^{\infty}\left[(k+1)(k)C_{k+1}-5(k+2)(k+1)C_{k+2}-[k+2]C_k\right]x^k=0 \checkmark$$

Problem 9 C 5/5 reoccurence relation and highest index is C_k will be

$$(k+1)(k)C_{k+1}-5(k+2)(k+1)C_{k+2}-[k+2]C_k=0 \checkmark$$

$$\therefore \quad -[k+2]C_k=-(k+1)(k)C_{k+1}+5(k+2)(k+1)C_{k+2}$$

$$C_k=\frac{+(k+1)(k)C_{k+1}+5(k+2)(k+1)C_{k+2}}{+[k+2]}$$

$$C_k=\frac{(k+1)(k)C_{k+1}+5(k+2)(k+1)C_{k+2}}{[k+2]}=\frac{2C_2+30C_3}{3}$$

5

Our highest index is C_{k+2} ✓

$$(k+1)(k)C_{k+1}-5(k+2)(k+1)C_{k+2}-[k+2]C_k=0 \checkmark$$

$$+5(k+2)(k+1)C_{k+2}=+(k+1)(k)C_{k+1}+(k+2)C_k$$

$$C_{k+2}=\frac{(k+1)(k)C_{k+1}+(k+2)C_k}{5(k+2)(k+1)}$$ OK $k\geq 1$ $k\geq 0$

Problem 9 d 5/5

Solving the first five terms which is C_0, C_1, C_2, C_3 & C_4

$C_0=C_0$ ✓

$C_1=C_1$ ✓

at $k=0$ $$C_{0+2} = \frac{(0+1)(0)C_{0+1} + (0+2)C_0}{5(0+2)(0+1)}$$

$$C_2 = \frac{(1)(0)C_1 + (2)C_0}{5(2)(1)}$$

$$C_2 = \frac{2C_0}{10} = \frac{1}{5}C_0$$

at $k=1$ $$C_{1+2} = \frac{(1+1)(1)C_{1+1} + (1+2)C_1}{5(1+2)(1+1)}$$

$$= \frac{2C_2 + 3C_1}{5(3)(2)}$$

$$= \frac{2C_2 + 3C_1}{30} = \frac{2\left(\frac{1}{5}\right)C_0 + 3C_1}{30}$$

$$C_3 = \frac{\frac{2}{5}C_0}{30} + \frac{3C_1}{30} = \frac{C_0}{75} + \frac{C_1}{10}$$

P.T.O for C_4

for C_4

at $k=2$

$$C_{2+2} = \frac{(2+1)(2)C_{2+1} + (2+2)C_2}{5(2+2)(2+1)}$$

$$C_4 = \frac{(3)(2)C_3 + (4)C_2}{5(4)(3)}$$

$$C_4 = \frac{6C_3 + 4C_2}{60}$$

$$C_4 = \frac{6\left(\frac{C_0}{75} + \frac{C_1}{10}\right) + 4\left(\frac{1}{5}\right)C_0}{60}$$ ✓

$$C_4 = \frac{\left(\frac{6C_0}{75} + \frac{6C_1}{10}\right) + \left(\frac{4}{5}\right)C_0}{60}$$

$$C_4 = \frac{\frac{60C_0 + 450C_1}{750}}{60} + \frac{\frac{4}{5}C_0}{60}$$ ✓

$$C_4 = \frac{60C_0 + 450C_1}{45000} + \frac{4}{300}C_0$$

$\therefore$ our $y = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + \ldots\ldots$

$$y = C_0 + C_1 x + \tfrac{1}{5} C_0 x^2 + \left(\frac{C_0}{75} + \frac{C_1}{10}\right) x^3 + \left(\frac{60 C_0 + 450 C_1}{45000} + \frac{4}{300} C_0\right) x^4 + \ldots$$

Usually we would factor out C_0 and C_1 but mine looks scary scary and I might get it wrong. So i will leave it at that okay! ☺

Problem 1(e) $y(0) = 1$, $y'(0) = 3$

$\frac{2}{3}$

Since $y(x) = C_0 + C_1 x + C_2 x^2 \ldots$

So our $C_0 = 1$ ✓

Then $y'(x) = C_1 + 2C_2 x^2 + \ldots\ldots$

then our $C_1 = 3$

to get the final 5 terms we will substitute C_1 and C_0 back into C_2, C_3 & C_4.

$C_1 = 3$, $C_0 = 1$

$$C_2 = \frac{1}{5} C_0 = \frac{1}{5} \times 1 = \frac{1}{5}$$

$$C_3 = \frac{C_0}{75} + \frac{C_1}{10} = \frac{1}{75} + \frac{3}{10} = \frac{235}{750} = \frac{47}{150}$$

$$C_4 = \frac{60 C_0 + 450 C_1}{45000} + \frac{4 C_0}{300}$$

P.T.O

$$C_4 = \frac{60C_0 + 450C_1}{45000} + \frac{4}{300}C_0 =$$

$$C4 = \frac{60(1) + 450(3)}{45000} + \frac{4}{300}(1)$$

$$C4 = \frac{60 + 1350}{45000} + \frac{4}{300}$$

$$= \frac{1410}{45000} + \frac{4}{300}$$

$$C4 = \frac{603\cancel{000}}{1350\cancel{000}} = \frac{603}{1350}$$

So our $C_0 = 1$, $C_1 = 3$, $C_2 = \frac{1}{5}$, $C_3 = \frac{47}{150}$, $C4 = \frac{603}{1350}$

$$y = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + \ldots$$

So our $y = 1 + 3x + \frac{1}{5}x^2 + \frac{47}{150}x^3 + \frac{603}{1350}x^4 + \ldots$

1 Overall story (a) we find our ρ which always on the interval of $(-x, x)$.. (b) we have our y, y' and y'' then we substitute into our equation $(x-5)y'' - xy' - 2y = 0$. having that we then result in shifting and factorizing out then equate to 0 (c) here is where we get C_k and the highest index which is C_{k+2} (d) we find our C_0, C_1, C_2, C_3 and C_4. (e) here we actually plug in our C_0 and C_1 back into C_2, C_3 & C_4 to get our new y.