THE APPLICATION OF BLACK-SCHOLES EQUATION IN FINANCIAL MODELING: THEORY AND EQUATIONS

MAT2680 – Differential Equations

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Contents

Essay

The Black-Scholes equation,

introduced by economists Fischer Black and Myron Scholes in 1973, has had a profound impact on the field of quantitative finance. This revolutionary equation provides a mathematical framework for pricing options and derivatives, enabling investors and financial institutions to make informed decisions regarding risk management, hedging strategies, and portfolio optimization. The Black-Scholes equation, combined with its underlying assumptions, has transformed the way we understand and approach financial markets.

1. The Black-Scholes Equation

The Black-Scholes equation is a partial differential equation that models the price evolution of financial derivatives over time. It takes into account several key variables, including the current price of the underlying asset (S), the time to expiration (t), the risk-free interest rate (r), the strike price of the option (K), and the volatility of the asset (σ). The equation assumes a continuous and efficient market, where prices follow a geometric Brownian motion and investors can borrow and lend money at the risk-free rate.

The basic form of the Black-Scholes equation is:

$$\frac{\partial C}{\partial t} + \frac{\left(\frac{1}{2}\right)\sigma^{2S^{2\partial^{2C}}}}{\partial S^2} + \frac{rS\partial C}{\partial S} - rC = 0$$

where C represents the price of a European call option on the underlying asset.

2. Option Pricing

One of the primary applications of the Black-Scholes equation is in option pricing. An option is a financial instrument that provides the holder with the right, but not the obligation, to buy (call option) or sell (put option) an underlying asset at a predetermined price (strike price) within a specified period. The Black-Scholes equation offers a method for calculating the fair value of options, considering the asset price, volatility, time to expiration, and risk-free interest rate.

The Black-Scholes model provides closed-form solutions for European call and put options. The formula for the price of a European call option (C) is:

$$C = S * N(d1) - Ke^{-rt} * N(d2)$$

where S is the current price of the underlying asset, K is the strike price, r is the risk-free interest rate, t is the time to expiration, N() is the cumulative distribution function of the standard normal distribution, and d1 and d2 are defined as:

$$d1 = \frac{\left(\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)t\right)}{\left(\sigma\sqrt{(t)}\right)d2} = d1 - \sigma * \sqrt{(t)}$$

Similarly, the price of a European put option (P) can be calculated using the put-call parity relationship:

$$P = Ke^{(-rt)N(-d2)} - SN(-d1)$$

These equations enable market participants to determine the fair value of options, assess their profitability, and compare different investment opportunities.

3. Risk Management and Hedging

The Black-Scholes equation plays a critical role in risk management and hedging strategies. Financial institutions utilize options to manage their exposure to market fluctuations, and the Black-Scholes equation aids in evaluating the potential risks associated with their portfolios.

To effectively manage risk, analysts use the Greeks, a set of sensitivity measures derived from the Black-Scholes equation. The Greeks quantify the impact of changes in underlying asset price (delta), time to expiration (theta), volatility (vega), risk-free interest rate (rho), and the rate of change of volatility (vanna and volga) on the option price.

For instance, delta (Δ) represents the change in the option price

In conclusion…

In conclusion, the Black-Scholes equation has revolutionized the field of quantitative finance, providing a framework for pricing options and derivatives and enabling investors and financial institutions to make informed decisions regarding risk management, hedging strategies, and portfolio optimization. The equation, combined with its underlying assumptions and the use of sensitivity measures such as the Greeks, has become an essential tool in financial modeling. Despite its limitations and criticisms, the Black-Scholes equation remains a cornerstone of modern finance, shaping the way we approach risk and uncertainty in financial markets.

References

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