Population Growth With Food Supply

By: Ken Mei

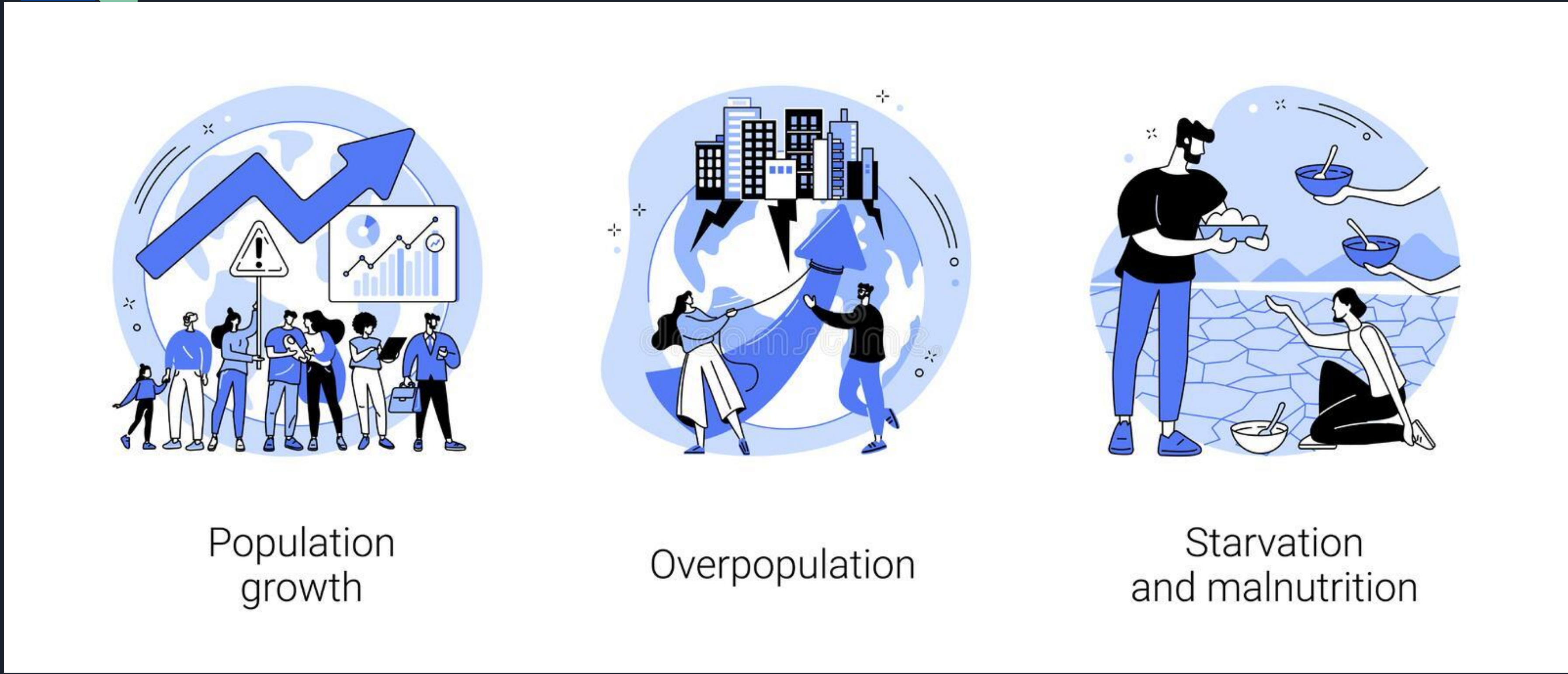
Introduction

The world population grows as time passes and as it increase the demand for food also increase. The more people there are, especially in poor countries with limited amounts of land and water, the fewer resources there are to meet basic needs. If basic needs cannot be met, development stalls and economies begin to unravel.

Poor countries attempts to increase food production and consumption are undermined by rapid population growth.

- Migration from rural to urban areas
- Unequal land distribution
- Shrinking household
- Deepening rural poverty
- Land degradation

Factor to Population Growth and Decay

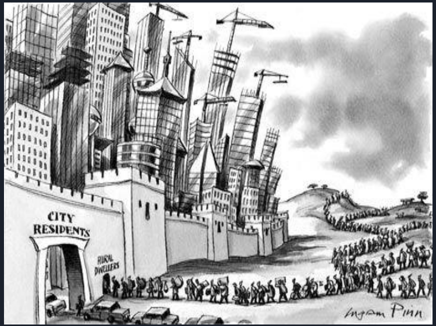


Issue with an growing population towards less developed countries

A countries ability to feed itself depends on three factors, availability of arable land, accessible water, and population pressures. In poor countries the attempts to increase food production and consumption are undermined due to the rapid population growth, migration from rural to urban areas, unequal land distribution, shrinking households, deepening rural poverty, and widespread land degradation. Lower birth rates to prevent rapid population growth and better management of the land and water resources, are necessary to prevent food shortages. The next two slides will show data the illustrates the population growth rate and agriculture production from 1950 - 2021.

Poverty

Land degradation



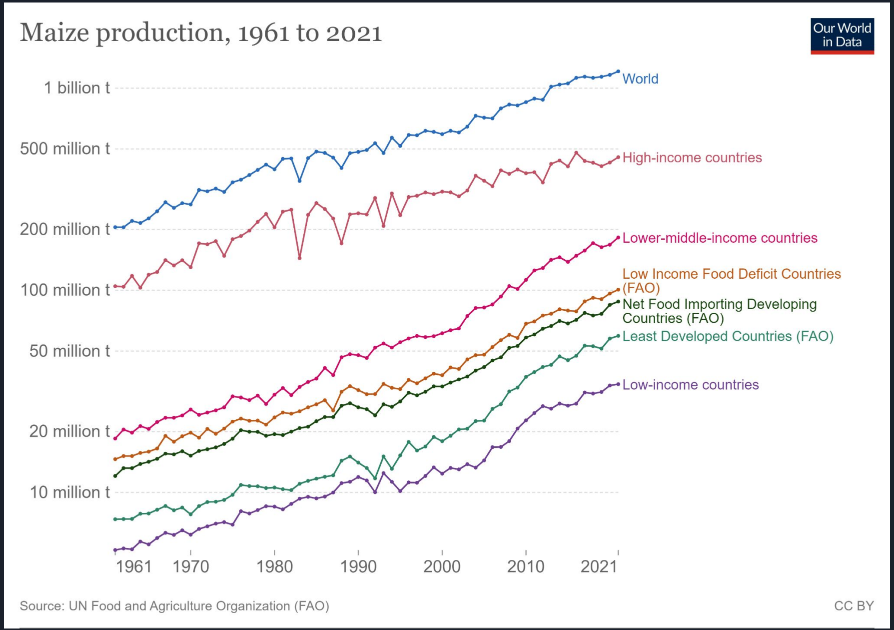
Migration from rural to urban areas

Accessible water

Statistics of Population Growth

- Less developed nations and lower income countries have a higher growth rate
- More developed nations and higher-income countries have a smaller population growth rate
- Upper-middle income countries saw a huge drop from the time period from 1990s to early 2000s

Statistics of Food Production



- There was a wide selection of foods like meat and crops that the world produces and they each could show a different pattern on a graph. I picked Maize (corn) to just provide a brief visual of what a certain crop production rate would look like in comparison to the world.
- Notice that more developed and higher income nations have higher production rates of a food then less developed and lower income nations.

Population Growth and Carrying Capacity

$P(t) = P_0e^{rt}$

- $P(t)$ represents the population at time t
- P_0 represents the initial population (population at time t = 0)
- r is a constant (r > 0) is called the growth rate

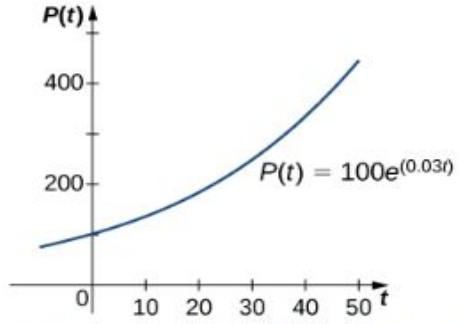


Figure 8.4.1: An exponential growth model of population.

This example (Figure 8.4.1) supports that the function P(t) = P_0e^{rt} satisfies the initial value problem (IVP), dP/dt = rP with P(0) = P_0 .

The differential equation (DE) dP/dt = rP has some issues. The left side is suppose to represents the rate at which the population increase or decrease. The right hand side is equal to an unchanged constant "r" multiplied by the current population.

The issue with this differential equation is that it's unrealistic in a real world setting. The formula is calculating the population growth without bound. It fails to consider limit factors such as birth rate, death rate, food supply, predators, etc.

The next slide shows an more accurate formula for calculating Population Growth with Food supply

Pierre Verhulst Logistic Differential Equation

Logistic Differential Equation:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

- K to denote carrying capacity
- Let r be a real number that represents the growth rate
- The function P(t) represents the population of this organism as a function of time t
- Constant P_0 represents initial population at time t = 0

Steps to Solving a Logistic Differential Equation Part 1

Step #2:

$$\frac{dP}{dt} = \frac{rP(K-P)}{K}$$

$$\frac{dP}{P(K-P)} = \frac{r}{K}dt.$$

$$\int \frac{K}{P(K-P)}dP = \int rdt.$$

$$\frac{K}{P(K-P)} = \frac{1}{P} + \frac{1}{K-P}.$$

- Step #1: Setting the right side equal to zero to get P = 0 and P= K as constant solutions. This step indicates there aren't any factors present that will make the population grow.
- Step #2: Rewrite the logistic differential equation in the form of dP/dt = rP(K-P)/K
- Step #3: Then multiply both sides by dt and divide both P(K-P).
- Step #4: Next, multiply both sides of the equation by K and integrate. The integral on the left hand side can be solved using partial fraction decomposition.

Step to Solving a Logistic Differential Equation Part 2

$$\int \frac{1}{P} + \frac{1}{K-P} dP = \int r dt$$

$$\ln|P| - \ln|K-P| = rt + C$$

$$\ln \left| \frac{P}{K-P} \right| = rt + C.$$

$$e^{\ln\left|\frac{P}{K-P}\right|} = e^{rt+C}$$

$$\left|\frac{P}{K-P}\right| = e^{C}e^{rt}.$$

$$\frac{P}{K-P} = C_1 e^{rt}.$$

- Step #6: Setting up the left side partial fraction decomposition and right side integral to solve.
- Step #7: We need to exponentiate on both sides to remove the natural logarithm
- Step #8: We defined $C_1 = e^c$ so that the equation becomes P / (K - P) $=C_1 e^{rt}$.

Step to Solving a Logistic Differential Equation Part 3

$$\begin{aligned} P &= C_1 e^{rt}(K - P) \\ &= C_1 K e^{rt} - C_1 P e^{rt} \\ P + C_1 P e^{rt} &= C_1 K e^{rt}. \end{aligned}$$

$$P(1 + C_1 e^{rt}) = C_1 K e^{rt}$$

$$P(t) = \frac{C_1 K e^{rt}}{1 + C_1 e^{rt}}.$$

$$\frac{P}{K - P} = C_1 e^{rt}$$

$$\frac{P_0}{K - P_0} = C_1 e^{r(0)}$$

$$C_1 = \frac{P_0}{K - P_0}.$$

- Step #9: To solve the equation for P(t0, first multiply both sides by (K - P) and collect the terms containing P so that the left hand side of the equation is $P + C_1 Pe^{rt} = C_1 K e^{rt}$.
- Step #10: Next factor P out from the left hand side and divide both sides by the factor of $(1 + C_1 e^{rt})$ to isolate P.
- Step #11: The last step is to determine the value of C_1. The simplest way to do it is substitute t = 0 and place P_0 in place of P in the equation and solve for C_1

Step to Solving a Logistic Differential Equation Part 4

$$P(t) = \frac{C_1 K e^{rt}}{1 + C_1 e^{rt}} = \frac{\frac{P_0}{K - P_0} K e^{rt}}{1 + \frac{P_0}{K - P_0} e^{rt}}$$

$$= \frac{\frac{P_0}{K - P_0} K e^{rt}}{1 + \frac{P_0}{K - P_0} e^{rt}} \cdot \frac{K - P_0}{K - P_0} = \frac{P_0 K e^{rt}}{(K - P_0) + P_0 e^{rt}}.$$

$$P(t) = \frac{P_0 K e^{rt}}{(K - P_0) + P_0 e^{rt}}$$

- Step #12: Then substitute the expression for C1 into the equation for P(t) from part 3
- Step #13: Now multiply the numerator and denominator of the right-hand side by (K - P_0) and then you can clean up the equation by simplifying.
- After simplifying you're left with the solution to the logistic differential equation of a initial value problem.
- P_0 is an initial population
- K is the carrying capacity
- r is for the growth rate

Conclusion

This project has reviewed the topic of population growth with food supply, through statistics and the steps to solving the differential equation that corresponds to this the topic. From the statistics alone we noticed for population growth rate for lower income and less developed countries have a higher population in comparison to higher income and more developed countries. This was the complete opposite when it came to agriculture production, we saw that high income and more developed countries had a higher production in comparison to lower income and less developed countries. Although such points were noticed from the date, the population around the globe is still increasing and so is the amount of food to compensate for the growing population.

Works Cited

"Global Food Explorer." *Our World in Data*, ourworldindata.org/explorers/global-food?tab=chart&yScale=log&facet=none&country=ZAF%3BhideControls~High-income%2Bcountries~Least%2BDeveloped%2BCountries%2B%28FAO%29~Low%2BIncome%2BFood%2BDeficit%2BCountries%2B%28FAO%29~Lower-middle-income%2Bcountries~Low-income%2Bcountries~Net%2BFood%2BImporting%2BDeveloping%2BCountries%2B%28FAO%29~OWID_WRL&Food=Maize%2B%28corn%29&Metric=Production&Per%2BCapita=false. Accessed 21 May 2023.

Libretexts, et al. "8.4: The Logistic Equation." *Mathematics LibreTexts*, 7 Sept. 2022, math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/08%3A_Introduction_to_Differential_Equations/8.04%3A_The_Logistic_Equation.

"Population & Demography Data Explorer." *Our World in Data*, ourworldindata.org/explorers/population-and-demography?facet=none&country=High-income%2Bcountries~Least%2Bdeveloped%2Bcountries~Less%2Bdeveloped%2Bregions~Low-income%2Bcountries~Lower-middle-income%2Bcountries~Upper-middle-income%2Bcountries~OWID_WRL~More%2Bdeveloped%2Bregions&hideControls=false&Metric=Population%2Bgrowth%2Brate&Sex=Both%2Bsexes&Age%2Bgroup=Total&Projection%2BScenario=None. Accessed 21 May 2023.

"Population Growth and the Food Crisis." *Population Growth and the Food Crisis*, www.fao.org/3/u3550t/u3550t02.htm.

Sources

- https://ourworldindata.org/explorers/global-food?tab=chart&yScale=log&facet=none&country=ZAF%3BhideControls~High-income+countries~Least+Developed+Countries+%28FAO%29~Low+Income+Food+Deficit+Countries+%28FAO%29~Lower-middle-income+countries~Low-income+countries~Net+Food+Importing+Developing+Countries+%28FAO%29~OWID_WRL&Food=Maize+%28corn%29&Metric=Production&Per+Capita=false
- https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/08%3A_Introduction_to_Differential_Equations/8.04%3A_The_Logistic_Equation
- https://ourworldindata.org/explorers/population-and-demography?facet=none&country=High-income+countries~Least+developed+countries~Less+developed+regions~Low-income+countries~Lower-middle-income+countries~Upper-middle-income+countries~OWID_WRL~More+developed+regions&hideControls=false&Metric=Population+growth+rate&Sex=Both+sexes&Age+group=Total&Projection+Scenario=None
- https://www.fao.org/3/u3550t/u3550t02.htm#:~:text=The%20more%20people%20there%20are,and%20economies%20begin%20to%20unravel