Linear homogenous

* $y' + p(x)y = 0$

$y = ce^{-\int p(x)dx}$

Ex → $y' + 2y = 0$

Linear Nonhomogenous

* $y' + p(x)y = f(x)$ $f(x) \neq 0$

① Solved the associated homogenous equation

② use variation of parameters to solve original equation.

③ Let y_1 be solution of $y = uy_1$

* $y' = u'y_1 + uy'_1$
* $u = \int \frac{f(x)}{y_1} dx$

5.3

* $y'' + p(x)y' + q(x)y = f(x)$, $f(x) \neq 0$
* $y = y_p + c_1 y_1 + c_2 y_2$

Integration Methods

* $\int u \cdot dv = u \cdot v - \int v \cdot du$
* $\int \frac{1}{1+y^2} = \tan^{-1}(y)$
* $\int \frac{1}{\cos^2(x)} = \tan(x)$

Seperable equations

* $h(y)y' = g(x)$

Bernomoulli Equations

* $y' + p(x)y = f(x)y^r$
* To Solve: ① y_1 is any solution of homogenous equation $y' + p(x)y = 0$

② Solve using variation of parameters

* $\frac{u'}{u^r} = f(x) y_1^{r-1}$

Set

$y' = uy_1$

$y' = u'y_1 + uy'_1$

Exact Equations

The equation * $M(x,y)dx + N(x,y)dy = 0$ is called $M_y = N_x$

Nonlinear homogenous

① Test wheather it is homogenous or not (if it depends only on $\frac{y}{x}$)

② $y = uy_1$

③ Plug $y = uy_1 = ux$ into $y' = f(x,y)$ to find out what u must satisfy

* $y = ux$

$y' = u'x + u$

* $\frac{u'}{q(u) - u} = \frac{1}{x}$
* $u = \frac{y}{x}$

5.2

* $ay'' + by' + cy = 0$ (a,b,c are constants)
* $ar^2 + br + c = 0$
* $y = c_1 y_1 + c_2 y_2$

Case 1

$y_1 = e^{r_1 x}$, $y_2 = e^{r_2 x}$ } $y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$ $(r_1 \neq r_2)$

Case 2

$y_1 = e^{rx}$, $y_2 = xe^{rx}$ } $y = c_1 e^{rx} + c_2 x e^{rx}$ $(r_1 = r_2)$

Case 3

$y_1 = e^{\lambda x}$, $y_2 = e^{\lambda x}\sin(\omega x)$ } $y = e^{\lambda x}(c_1\cos(\omega x) + c_2\sin(\omega x))$

5.7

* $y_p = u_1 y_1 + u_2 y_2$
* $u'_1 y_1 + u'_2 y_2 = 0$

5.4

* $ay'' + by' + cy = e^{\alpha x} G(x)$

Laplace Transform

* $\mathcal{L}(f') = s\mathcal{L}(f) - f(0)$
* $\mathcal{L}(f'') = s^2 \mathcal{L}(f) - f'(0) - sf(0)$

① apply $\mathcal{L}$ to both sides

② Solve for $Y(s)$

③ Want to apply $\mathcal{L}^{-1}$ to both sides.

5.6

* Ex → Given that y_1 is a solution of $x^2y'' + 5xy' - 5y = 0$

① $y = uy_1$

$y = ux$

$y' = u + u'x$

$y'' = 2u' + u''x$

② $LS = x^2(ux)'' + 5x(ux)' - 5(ux)$

$= x^3u'' + 7x^2u'$

$RS = 0$

$x^3u'' + 7x^2u' = 0$

Let $w = u'$

$w' = u''$

③ $x^3 w' + 7x^2 w = 0$

First order differential

$w = cx^{-7}$

$w = u'$

$u' = cx^{-7}$

$u = c_1 x^{-6} + c_2$

$y = ux$

$y = (c_1 x^{-6} + c_2)x$

$\boxed{y = c_1 x^{-5} + c_2 x}$