P1

$(x-5)y'' - xy' - 2y = 0$

a) Since it center at 0

$x - 5 = 0$
$+5 \quad +5$

$x = 5$

$(0+5, 0-5) = \boxed{(5,-5)}$

b)

$y = \sum_{k=0}^{\infty} c_k x^k$

$y' = \sum_{k=0}^{\infty} k c_k x^{k-1}$

$y'' = \sum_{k=0}^{\infty} k(k-1) c_k x^{k-2}$

$LS = (x-5)\sum_{k=0}^{\infty} k(k-1) c_k x^{k-2} - x\sum_{k=0}^{\infty} k c_k x^{k-1} - 2\sum_{k=0}^{\infty} c_k x^k$

$= (x-5)\sum_{k=0}^{\infty} k(k-1) c_k x^{k-2} - x\sum_{k=0}^{\infty} k c_k x^{k-1} - 2\sum_{k=0}^{\infty} c_k x^k$

$= \underset{A}{\sum_{k=0}^{\infty} k(k-1) c_k x^{k-1}} - \underset{B}{\sum_{k=0}^{\infty} 5k(k-1) c_k x^{k-2}} - \underset{C}{\sum_{k=0}^{\infty} k c_k x^k} - \underset{D}{\sum_{k=0}^{\infty} 2 c_k x^k}$

✓ ✓

Ⓐ $\sum_{k=0}^{\infty} k(k-1)c_k x^{k-1} = \underbrace{\overset{k=0}{0(0-1)c_0 x^{0-1}}}_{0} + \sum_{k=1}^{\infty} k(k-1)c_k x^{k-1} = \sum_{k=0}^{\infty} (k+1)k\, c_{k+1} x^k$

Ⓑ $\sum_{k=0}^{\infty} 5k(k-1)c_k x^{k-2} = \underbrace{\overset{k=0}{(5)(0)(0-1)c_0 x^{0-2}}}_{0} + \underbrace{\overset{k=1}{(5)(1)(1-1)c_1 x^{1-2}}}_{0} + \sum_{k=2}^{\infty} 5k(k-1)c_k x^{k-2}$

shift up 2

$= \sum_{k=0}^{\infty} 5(k+2)(k+1)c_{k+2} x^k$

$= \sum_{k=0}^{\infty} (k+1)k\, c_{k+1} x^k - \sum_{k=0}^{\infty} 5(k+2)(k+1)c_{k+2} x^k - \sum_{k=0}^{\infty} k c_k x^k - \sum_{k=0}^{\infty} 2c_k x^k$

$= \sum_{k=0}^{\infty} \left[(k+1)k\, c_{k+1} - 5(k+2)(k+1)c_{k+2} - k c_k - 2c_k\right] x^k$

$$\boxed{= \sum_{k=0}^{\infty} \left[(k+1)k\, c_{k+1} - 5(k+2)(k+1)c_{k+2} - (k+2)c_k\right] x^k = 0}$$

c) $(k+1)k\, c_{k+1} - 5(k+2)(k+1)c_{k+2} - (k-2)c_k = 0$

$+5(k+2)(k+1)c_{k+2} \qquad +5(k+2)(k+1)c_{k+2}$

$$\frac{5(k+2)(k+1)c_{k+2}}{5(k+2)(k+1)} = \frac{(k+1)k\, c_{k+1} - (k-2)c_k}{5(k+2)(k+1)}$$

$$c_{k+2} = \frac{(k+1)k\, c_{k+1} - (k-2)c_k}{5(k+2)(k+1)} \qquad k \le 1$$

D) $c_0 = c_0$

$c_1 = c_1$

$$K=0 \quad c_{0+2} = \frac{(0+1)(0)c_{0+1} + (0+2)c_0}{5(0+2)(0+1)}$$

$$c_2 = \frac{(1)(0)c_1 + (2)c_0}{5(2)(1)}$$

$$c_2 = \frac{2}{10}c_0 = \frac{1}{5}c_0$$

$$c_2 = \frac{1}{5}c_0$$

$$K=1 \quad c_{1+2} = \frac{(1+1)(1)c_{1+1} + (1+2)c_1}{5(1+2)(1+1)}$$

$$= \frac{2c_2 + 3c_1}{5(3)(2)} = \frac{2c_2 + 3c_1}{30} = \frac{2\left(\frac{1}{5}c_0\right) + 3c_1}{30}$$

$$= \frac{\frac{2}{5}}{30}c_0 + \frac{3}{30}c_1 = \frac{1}{75}c_0 + \frac{1}{10}c_1$$

$$c_3 = \frac{1}{75}c_0 + \frac{1}{10}c_1$$

$$k=2 \quad c_{2+2} = \frac{(2+1)(2)c_{2+1} + (2+2)c_2}{5(2+2)(2+1)}$$

$$c_4 = \frac{(3)(2)c_3 + (4)c_2}{5(4)(3)} = \frac{6}{60}c_3 + \frac{4}{60}c_2$$

$$C_4 = \frac{6\left(\frac{1}{75}C_0 + \frac{1}{10}C_1\right)}{60} + \frac{4\left(\frac{1}{5}\right)C_0}{60}$$

$$C_4 = \frac{\left(\frac{6C_0}{75} + \frac{6C_1}{10}\right)}{60} + \frac{\left(\frac{4}{5}\right)C_0}{60}$$

$$C_4 = \frac{\frac{60C_0 + 450C_1}{750}}{60} + \frac{\frac{4}{5}C_0}{60}$$

$$C_4 = \frac{60C_0 + 450C_1}{45000} + \frac{4}{300}C_0$$

$$y = C_0 + C_1x + C_2x^2 + C_3x^3 + C_4x^4 +$$

$$\boxed{\begin{aligned} y = C_0 + C_1x + \frac{1}{5}C_0x^2 + \left(\frac{1}{75}C_0 + \frac{1}{10}C_1\right)x^3 \\ + \left(\frac{60C_0 + 450C_1}{45000} + \frac{4}{300}C_0\right)x^4 + \dots \end{aligned}}$$

e) $y(0) = 1$, $y'(0) = 3$

$$y(x) = C_0 + C_1x + C_2x^2$$

$$C_0 = 1$$

$$y'(x) = C_1 + 2C_2x^2 +$$

$$c_1 = 3$$

$$c_1 = 3 \quad , \quad c_0 = 1$$

$$c_2 = \frac{1}{5} c_0 = \frac{1}{5} \times 1 = \frac{1}{5}$$

$$c_3 = \frac{c_0}{75} + \frac{c_1}{10} = \frac{1}{75} + \frac{3}{10} = \frac{235}{750} = \frac{47}{150}$$

$$c_4 = \frac{60 c_0 + 450 c_1}{45000} + \frac{4}{300} c_0 =$$

$$= \frac{60(1) + 450(3)}{45000} + \frac{4}{300}(1)$$

$$= \frac{60 + 1350}{45000} + \frac{4}{300}$$

$$= \frac{1410}{45000} + \frac{4}{300} = \frac{603}{1350}$$

$c_0 = 1$

$c_1 = 3$

$c_2 = 1/5$

$c_3 = \frac{47}{150}$

$c_4 = \frac{603}{1350}$

$$y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \dots$$

$$y = 1 + 3x + 1/5\, x^2 + \frac{47}{150} x^3 + \frac{603}{1350} x^4$$