Project 3 Final:

Differential Equations & The Predator-Prey Model

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Differential equations serve as a mathematical tool for describing the behavior of the natural world. Many laws that govern natural phenomena involve relationships that describe the rates at which events occur. To express these laws mathematically, these relationships are transformed into equations, with the rates represented as derivatives. Thus, any equations that involve derivatives are known as differential equations. Differential Equations has been used to analyze and model various different types of phenomena and scientific studies. One great example of when differential equations is applied to a real life phenomena is Predator-Prey model. This model is used to capture the dynamic interactions between two species within their ecosystem.

Using differential equations scientists are able to learn and gain a deeper understanding of the behavior of these two species and how their populations change as time passes. Using the following pair of equations, known as the Lotka-Volterra Equations, mathematicians employ a system of ordinary differential equations in order to represent the population dynamics of the predator and prey species.

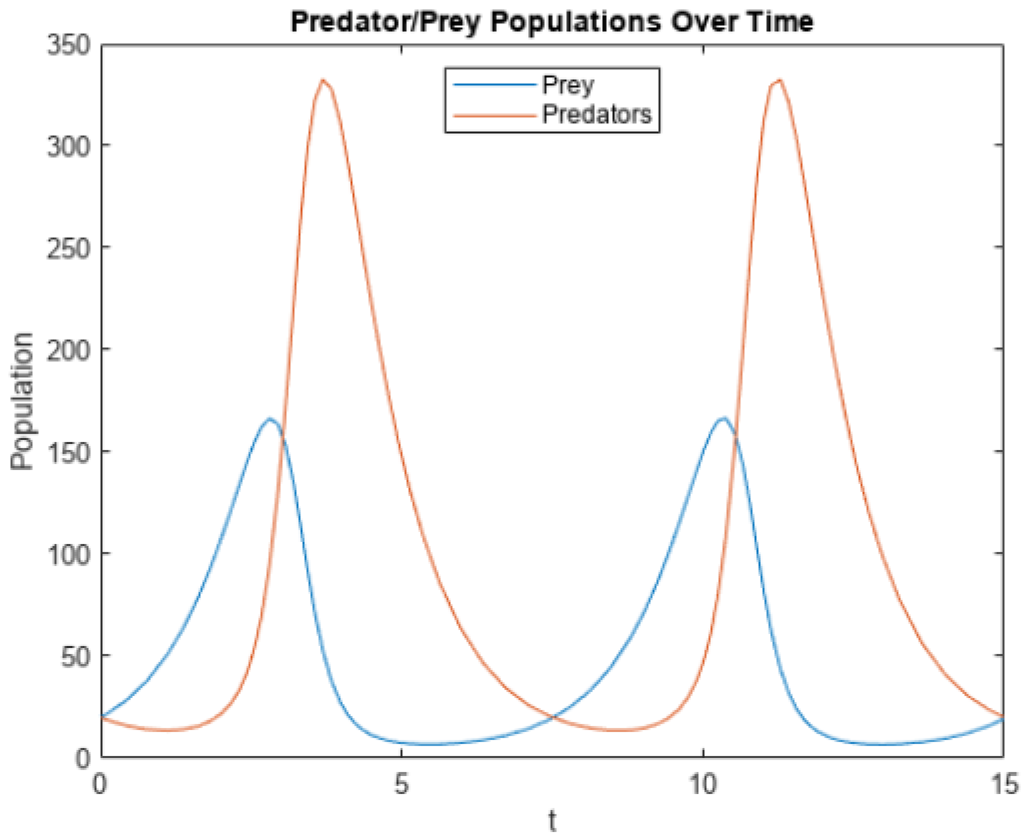
- $\frac{dx}{dt} = \alpha x - \beta xy$
- $\frac{dy}{dt} = \delta xy - \gamma y$

- The variable x represents the population density of the prey (Ex. Rabbits per square kilometer)
- The variable y represents the population density of the predator (Ex. Foxes per square kilometer)
- $\frac{dx}{dt}$ and $\frac{dy}{dt}$ represents the instantaneous growth rates of the predator and prey populations.
- t, represents time.
- α *and* β are the prey's parameters and they describe the maximum prey growth rate per capita (α) and the effect that the predators have on the prey's population growth rate (β).
- δ *and* γ describes the predator's death rate per capita (γ) and the effect that the prey has on the predator's population growth rate (δ).
- All parameters are positive and real.

The solution to this set of differential equations is both deterministic and continuous.

("Lotka–Volterra Equations." *Wikipedia*, Wikimedia Foundation, 5 May 2023, en.wikipedia.org/wiki/Lotka%E2%80%93Volterra_equations. Accessed 6 May 2023.)

Predator-Prey Model Plot example:



These coupled differential equations capture the rate of change of the populations with respect to time and incorporate the interactions between the species.

This model of a system of differential equations was first created in the 1920s. A biologist named Humberto D'Ancona conducted a statistical analysis of the species trade at 3 different italian ports. His study at the time focused on fish species from 1914 to 1923, and he made quite a remarkable discovery. D'Ancona believed that by studying the predator-prey relationships between sharks, rays, and other fish he

was able to see their true natural state. Once he realized the significance of his discovery her sought out his father-in-law, Vito Volterra who was an esteemed mathematician at the time, to help analyze and draw conclusions based on the data that he collected from his research. Volterra, after several weeks of developing models for the interactions between two species coincidentally at the time arrived at similar conclusions as an American biologist and mathematician Alfred J. Lotka. The Lotka-Volterra system of equations was formed.
("Analyzing Predator-Prey Models Using Systems of Ordinary Linear Differential Equations." Https://Opensiuc.Lib.Siu.Edu/, 8 May 2011, opensiuc.lib.siu.edu/cgi/viewcontent.cgi?article=1349&context=uhp_theses. Accessed 7 May 2023.)

The Lotka-Volterra system of equations enable researchers to explore the dynamics of the interactions between predator and prey. Using mathematical analysis, some essential concepts were able to be derived; Equilibrium Points, Population Cycles, and Stability Analysis. To start off, Equilibrium Points are points that occur on a graph when both the predator and the prey populations are stablilized. We are able to get these points by setting both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ to zero and then solving the equations. This allows scientists to gain insight on how the dynamics of the predator-prey system works in the long term. Next concept is Population Cycles, which is where the predator and prey populations would oscillicate over time. This would occur due to the interactions between the predator and prey species. Finally, Stability Analysis plays a significant role in how we understand the long-term behavior of the predator prey system. When stable equilibrium is attained, it indicates to us that there is a balance between the predator and prey populations. Whereas, if there is unstable equilibria present lead to fluctuations in population which can eventually lead to the extinction of either or both species. In the predator-prey model, it is assumed that the prey species have an abundant food supply and this leads to their population to grow exponentially unless they are preyed upon by predators. If either the prey population or predator population is equal to zero, no predation can occur. Thus, the equation for the population growth highlights that the rate of change of the prey populus is determined by the growth rate minus the rate at which it is consumed by predators. Therefore, the equation expresses that the rate of change of the predator population depends solely on their consumption of prey.

In conclusion, differential equations serve as a powerful mathematical tool for describing natural phenomena and relationships in the natural world. By expressing these relationships as equations involving derivatives, scientists and mathematicians can analyze and model various phenomena and scientific studies.

One notable application is the Predator-Prey model, which uses differential equations to capture the dynamic interactions between two species within an ecosystem. The Lotka-Volterra equations, developed in the 1920s, are a prime example of a system of differential equations used to represent the population dynamics of predator and prey species. These equations incorporate parameters that describe the maximum growth rate, predation rate, and mortality rate of the species. Through mathematical analysis, researchers can explore important concepts such as equilibrium points, population cycles, and stability analysis, providing valuable insights into the long-term behavior of predator-prey systems. The predator-prey model overall is one significant way that scientists and researchers can use differential equations and apply it to real-life phenomena.

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