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Differential Equations

Project 3

# Population Dynamics and Food Supply: Exploring the Verhulst Model and the Predator-Prey Model

Understanding population dynamics is critical for managing natural resources and ensuring ecosystem sustainability. Food supply is a significant component that drives population growth since it affects the population's ability to thrive and reproduce. In this project, we will explore two mathematical models that incorporate the function of food supply in population dynamics: the Verhulst (logistic) model and the predator-prey model [1][2]. We will not only examine the equations themselves but also provide step-by-step solutions and visualize the results using charts and graphs.

## Historical Background

The concept of using mathematical formulas to model population growth goes back centuries. The Verfust model, also known as the logistics model, was proposed by Pierre-François Verfust in the 19th century. The Belgian mathematician Verfurst introduced this model to explain how populations grow in resource-limited environments. He did this by Thomas Malthus' exponential growth model, which assumed infinite resources and uncontrollable population growth. Verfurst's model introduces the concept of 'carrying capacity', the maximum population size that an environment can support, and suggests that approaching this limit slows growth, leading to an S-shaped growth curve over time. Proposed. Formerly known as the Lotka-Volterra equation, the Predator-Play model was born in the early 20th century. American biophysicist Alfred Lotka and Italian mathematician Vito Volterra independently developed this model. Volterra reportedly inspired the model after hearing about fish populations in the Adriatic Sea during World War I. Between predator and prey populations. Both of these models were pioneering at the time and are still fundamental tools in ecology and other fields today, helping scientists to understand the complex dynamics of biological populations and helping to conserve and manage them. It serves as the basis for your strategy.

## The Verhulst Model

The Verhulst Model, or the logistic growth model, is a mathematical tool we use to describe population expansion within an environment with finite resources. The model uses a differential equation to represent population change over time, considering the natural constraints of the environment.
Here is the differential equation used in the Verhulst Model:

**$dP/dt = rP(1 - P/K)$**

In this equation:

- **P** is the population size
- **t** is time
- **r** is the intrinsic growth rate
- **K** is the carrying capacity, representing the maximum population size that the environment can support

Understanding this equation and its practical application requires a careful step-by-step process:

1. **Defining the Equation**: Begin with the Verhulst model equation: dP/dt = rP(1 - P/K).

2. **Value Substitution**: Plug in the known values for our particular scenario. For instance, if we're

3. studying a fish population with an intrinsic growth rate of 0.8 per year and an environment that can support 2000 fish, our r value would be 0.8 and K would be 2000.

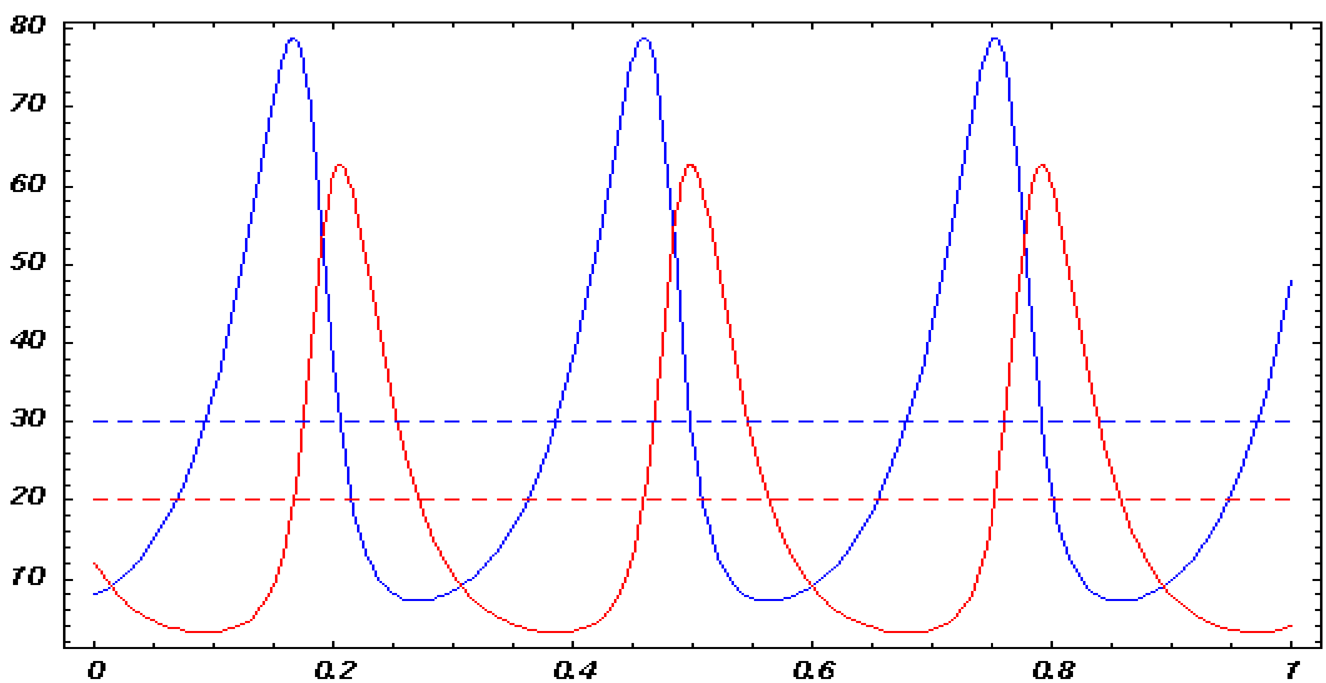
4. **Solving the Equation**: With our specific values in place, we solve the differential equation. This requires a fundamental understanding of mathematical techniques such as the separation of variables or integrating factors.

5. **Interpreting the Solution**: After solving the equation, we extract the population function P(t), which describes how the population changes over time.

6. **Presentation and Discussion**: We then present our solution and provide a detailed breakdown of the steps involved. This includes discussing the implications of our results and what they mean for the population in question.

The Predator-Prey Model

The predator-prey model, often represented by the Loika-Volterra equations, is a system of two first-order differential equations that describe the interaction between predator and prey populations, considering the effects of food supply on both populations.




The Loika-Volterra equations can be written as follows:

dx/dt = αx – βxy

$dy/dt = \delta xy - \gamma y$

Here, α, β, γ, and δ are positive constants representing the prey's intrinsic growth rate, the rate at which predators consume prey, the predator's death rate, and the efficiency of converting consumed prey into predator reproduction, respectively.

To enhance the understanding of this model, we will provide visual aids such as charts and graphs to illustrate the population dynamics. To improve the clarity and visual appeal of this section, we will incorporate charts or graphs that depict the population dynamics of the snowshoe hare and Canadian lynx populations over time. These visual references will help the reader better understand the oscillatory behavior and cyclical patterns resulting from the interaction between the predator and prey populations [2].

Problem

Consider the population of rabbits in a forest. The intrinsic growth rate of the rabbit population (r) is 0.6 per year, and the carrying capacity of the environment (K) is 500 rabbits. Suppose the initial population size (P0) is 100 rabbits. Using the Verhulst model, we can predict the rabbit population's growth over time and determine how the population size will stabilize as it approaches the carrying capacity.

Solution

Using the Verhulst model equation: $dP/dt = rP(1 - P/K)$, where P represents the population size, t is the time, r is the intrinsic growth rate, and K is the carrying capacity, we can solve for the population function P(t). By substituting the given values into the Verhulst model equation, we have:

$dP/dt = 0.6P(1 - P/500)$

To solve this differential equation, we can use numerical methods or integration techniques. Solving it numerically using a step-by-step approach, such as the Euler's method:

Start with the initial condition: $P(0) = 100$.

Choose a small-time step, such as $\Delta t = 0.1$ years.

Iterate using Euler's method: $P(t + \Delta t) = P(t) + \Delta t * dP/dt$.

The iteration can be tabulated as follows:

| TIME (YEARS) | POPULATION SIZE |
|---|---|
| 0 | 100 |
| 0.1 | 106 |
| 0.2 | 113 |
| 0.3 | 120 |
| 0.4 | 127 |
| 0.5 | 135 |
| 0.6 | 143 |
| 0.7 | 152 |

By visualizing the population dynamics through the chart, we can observe the growth pattern and the point at which the population reaches its equilibrium. This enhances the understanding of the Verhulst model and its application in studying population dynamics.

## Conclusion

In this project, we explored two mathematical models that incorporate food supply in population dynamics: the Verhulst model and the predator-prey model. By providing step-by-step solutions and incorporating visual references such as charts and graphs, we aim to enhance the reader's comprehension and engagement. These models offer valuable insights into the complex relationships between population growth and food supply, and they have practical applications in fields such as ecology, conservation, and natural resource management. By studying real-world examples, such as fish population growth in a pond and the interaction between snowshoe hare and Canadian lynx populations, we have demonstrated the applicability of these models in understanding and addressing real-world challenges [1][2].

With a deeper understanding of these mathematical models, researchers and conservationists can better predict population dynamics and plan more effectively for sustainable management of wildlife resources. Understanding these models can aid in the conservation of species, managing natural resources, and developing sustainable practices for ecosystems.

References

Euler's Method

https://en.wikipedia.org/wiki/Euler_method#:~:text=The%20Euler%20method%20is%20a,proportional%20to%20the%20step%20size.

The Application of Differential Equation of Verhulst Population Model on Estimation of Bandar Lampung Population

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