Formula Sheet/Cheat Sheet

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Constant Coefficient Homogeneous Basic

$y'' - 6y' + 8y = 0$

$r^2 - 6r + 8 = 0$

$r^2 - 2r - 4r + 8 = 0$

$(r-4)(r-2) = 0$

$r_1 = 4$, $r_2 = 2$

General Solution: $y(t) = Ae^{r_1 t} + Be^{r_2 t}$

$y(t) = Ae^{4t} + Be^{2t}$, Initial conditions

$y(0) = -14$, $y'(0) = -38$ } $-38 = 4A + 2B \rightarrow (2)$

$-14 = A + B \rightarrow (1)$ } Solve for (1) and (2)

$y'(t) = 4Ae^{4t} + 2Be^{2t}$ } $A = -5$, $B = -9$

Constant Coefficient Homogeneous Complex

$y'' + 12y' + 72y = 0 \,] \rightarrow r^2 + 12r + 72 = 0$, $a = 1$, $b = 12$, $c = 72$

$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(12) \pm \sqrt{(12)^2 - 4(1)(72)}}{2(1)} = \frac{-12 \pm 12i}{2}$

$\lambda + \nu i$

$= -6 \pm 6i$, $\lambda = -6$, $\nu = 6$, $r_1 = -6 + 6i$, $r_2 = -6 - 6i$

General Solution: $Ae^{-6t}\cos(6t) + Be^{-6t}\sin(6t)$

$y(0) = 8$, $y'(0) = 6$, $Ae^{-6(0)}\cos(6(0)) + Be^{-6(0)}\sin(6(0)) = 8$

$A = 8$, $y'(t) = \frac{d}{dt}(8e^{-6t}\cos(6t)) + \frac{d}{dt}(Be^{-6t}\sin(6t))$

Use Product Rule: $(f \cdot g)' = f' \cdot g + f \cdot g'$, to solve for $y'(t)$ then apply $y'(0) = 6$ to solve for B then substitute the values for A and B back into the general soln to get the particular solution

Constant Coefficient Homogeneous Repeated: $y'' - 12y' + 36y = 0$

$r^2 - 12r + 36 = 0 \,] \rightarrow (r-6)(r-6) = 0$, $r_1 = 6$, $r_2 = 6$

$y_1 = e^{r_1 x}$, $y_2 = xe^{r_1 x} \,] \rightarrow y_1 = e^{6t}$, $y_2 = te^{6t}$, General Solution: $\rightarrow$

$\rightarrow Ae^{6t} + Bte^{6t}$, Initial conditions $y(0) = 3$, $y'(0) = 10$

$y(0) = Ae^{6(0)} + B(0)e^{6(0)} = 3$, $A = 3 \,] \rightarrow y(t) = 3e^{6t} + Bte^{6t}$

$y'(t) = \frac{d}{dt}(3e^{6t}) + \frac{d}{dt}(Bte^{6t}) = y'(t) = 18e^{6t} + B(e^{6t} + 6te^{6t})$

$y'(0) = 18e^{6(0)} + B(e^{6(0)} + 6(0)e^{6(0)}) = 10$, $B = -8$, Particular Soln $= 3e^{6t} - 8te^{6t}$

Nonhomogeneous Linear (Case A)

$y'' + 11y' + 28y = -28e^{-3t}$

$r^2 + 11r + 28 = 0 \,] \rightarrow (r+7)(r+4) = 0$

$r_1 = -7$, $r_2 = -4$, $y_1 = e^{-7t}$, $y_2 = e^{-4t}$

General Solution: $Ae^{-7t} + Be^{-4t}$

Case A $= r_1, r_2 \neq \alpha$

Case B $= r_1$ or $r_2 = \alpha \cdot t$

Case C $= r_1 = r_2 = \alpha \cdot t^2$

$Y(t) = \frac{1}{(r+7)(r+4)}(-28e^{-3t})$

Plug $\alpha = -3$ into "r" for $Y(t)$ to solve $] \rightarrow -7e^{-3t}$

General Solution: $Ae^{-7t} + Be^{-4t} - 7e^{-3t}$, then find $y'(t)$ to solve for the initial conditions $y(0) = 10$ and $y'(0) = -74$ to get particular Soln: $9e^{-7t} + 8e^{-4t} - 7e^{-3t}$

Nonhomogeneous Linear (Case B)

$y'' - 11y' + 30y = 7e^{5t}$

$r^2 - 11r + 30 = 0$

$(r-5)(r-6) = 0$

$r_1 = 5$, $r_2 = 6$

$y_1 = e^{5t}$, $y_2 = e^{6t}$

General Solution: $Ae^{5t} + Be^{6t}$

Case A $= r_1, r_2 \neq \alpha$

Case B $= r_1$ or $r_2 = \alpha \cdot t$

Case C $= r_1 = r_1 = \alpha \cdot t^2$

$Y = Ate^{5t}$

$y' = A \cdot (\frac{d}{dt}(t \cdot e^{5t}))$

$= A(e^{5t} + 5te^{5t})$

$y'' = A(\frac{d}{dt}(e^{5t} + 5te^{5t}))$

$= A(10e^{5t} + 25te^{5t})$

Plug y'', y' into the equation and after canceling out like terms left with

$\frac{-Ae^{5t}}{-e^{5t}} = 7e^{5t} \,] \rightarrow A = -7$

$Y(t) = -7te^{5t}$

$y(t) = -7te^{5t} + Ae^{5t} + Be^{6t}$ general solution

for the particular soln

$y(0) = 10$, $y'(0) = 46$, find $y'(t)$ and solve the 2x2 system of eq'n

Nonhomogeneous Linear (Case C): $y'' - 8y' + 16y = 0$

$r^2 - 8r + 16 = 0 \,] \rightarrow (r-4)(r-4) = 0 \,] \rightarrow r_1 = 4$, $r_2 = 4$

General Solution: $Ae^{4t} + Bte^{4t}$ $\quad y_1 = e^{4t}$, $y_2 = te^{4t}$

$Y_p = At^2e^{4t}$, $y' = A(\frac{d}{dt}(t^2e^{4t}))$

$y' = A(2te^{4t} + 4t^2e^{4t})$

$y'' = A(\frac{d}{dt}(2te^{4t} + 4t^2e^{4t}))$

Case A $= r_1, r_2 \neq \alpha$

Case B $= r_1$ or $r_1 = \alpha \cdot t$

Case C $= r_1 = r_2 = \alpha \cdot t^2$

Plug into the eq'n and cancel out like terms to solve for A to get $A = -8$ so $Y(t) = -8t^2e^{4t}$

General Solution: $Y(t) + Y_h(t) = -8t^2e^{4t} + Ae^{4t} + Bte^{4t}$

Solve the particular Solution $y(0) = -2$, $y'(0) = -13$

Reduction of order: $x^2y'' - 7xy' + 16y = 0$ has x^4 as a solution

Applying reduction order we set $y = ux^4$

$y' = \frac{d}{dx}(u \cdot x^4) = u'x^4 + 4ux^3$

$y'' = \frac{d}{dx}(u'x^4 + 4ux^3) = u''x^4 + 8u'x^3 + 12ux^2$

Plug y' and y'' into equation and simplify

$x^5(xu'' + u') = 0$, sub $v = u'$ and $w' = u'' \,] \rightarrow xw' + w = 0$

$\frac{dv}{dx} = \frac{-w}{x} \,] \rightarrow \frac{dw}{v} = \frac{-dx}{x} \rightarrow -\ln(x) + \log(a)$

$wx = a$, $w = \frac{a}{x} \rightarrow u' = \frac{a}{x} \rightarrow \frac{du}{dx} = \frac{a}{x} \rightarrow \int du = \int a\frac{dx}{x}$

$u = a\ln(x) + b$, General Solution is: $y = u \cdot x^4$

$y = (a\ln(x) + b)x^4 \rightarrow y = ax^4\ln(x) + bx^4$

$\frac{d}{dx}e^x = e^x$, $\frac{d}{dx}\ln(x) = \frac{1}{x}$, $\frac{d}{dx} - \cos(x) = \sin(x)$, $\frac{d}{dx} - \sin(x) = -\cos(x)$

$\frac{d}{dx}\cot(x) = -\csc^2(x)$, $\frac{1}{\cos^2(x)} = \sec^2(x)$, $\frac{1}{\sin^2(x)} = \csc^2(x)$, $\frac{1}{\tan^2(x)} = \cot^2(x)$

$\frac{d}{dx}\sin(x) = \cos(x)$, $\frac{d}{dx}\cos(x) = -\sin(x)$, $\frac{d}{dx}\tan(x) = \sec^2(x)$, $\frac{d}{dx}\sec(x) = \sec(x) \cdot \tan(x)$

$\frac{d}{dx}\csc(x) = -\cot(x) \cdot \csc(x)$, $\int \sin(x)dx = -\cos(x) + C$, $\int \cos(x)dx = \sin(x) + C$

$\int \tan(x)dx = -\ln|\cos(x)| + C$, $\int \sec^2(x)dx = \tan(x) + C$, $\int \tan(x) \cdot \sec(x)dx = \sec(x) + C$

$\int \csc^2(x)dx = -\cot(x) + C$, $\int \csc(x)dx = \ln|\tan(\frac{x}{2})| + C$, $\int \cot(x)dx = \ln|\sin(x)| + C$

Int by Parts: $\int u \cdot dv = u \cdot v - \int v\,du$, Chain Rule: $\frac{d}{dx}e^{g(x)} \cdot g'(x)$, $\int \frac{1}{\cos^2(x)}dx = \tan(x) + C$

Product Rule: $(f \cdot g)' = f' \cdot g + f \cdot g'$, $\cot = \frac{\cos\theta}{\sin\theta}$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\int \frac{1}{\sin^2(x)}dx = -\cot(x) + C$, $\int \frac{1}{\tan^2(x)}dx = -x - \cot(x) + C$