

Section 6.1 Spring Problems 1

1. C/G An object stretches a spring 4 inches in equilibrium. Find and graph its displacement for $t > 0$ if it's initially displaced 36 inches above equilibrium and given a downward velocity of 2 ft/s.

Given

$$C=0 \quad \Delta l = \frac{4}{12} = \frac{1}{3}$$

$$y(0) = \frac{36}{12} = 3$$

$$F=0 \quad g=32$$

$$y'(0) = -2$$

$$m y'' + k y = 0$$

$$\frac{k}{m} = \frac{g}{\Delta l} = \frac{32}{1/3} = 96$$

$$y'' + \frac{k}{m} y = 0$$

Characteristic Eq'n

$$y'' + 96 y = 0$$

$$r^2 + 96 = 0$$

$$r = \pm 4\sqrt{6}i$$

$$y(t) = C_1 \cos(4\sqrt{6}t) + C_2 \sin(4\sqrt{6}t) \leftarrow \text{general sol'n}$$

$$y'(t) = -4\sqrt{6} C_1 \sin(4\sqrt{6}t) + 4\sqrt{6} C_2 \cos(4\sqrt{6}t)$$

$$y(0) = C_1 \overset{1}{\cos(0)} + C_2 \overset{0}{\sin(0)} = 3$$

$$C_1 = 3$$

$$y'(0) = -4\sqrt{6} C_1 \overset{0}{\sin(0)} + 4\sqrt{6} C_2 \overset{1}{\cos(0)} = -2$$

$$4\sqrt{6} C_2 = -2 \rightarrow C_2 = -\frac{1}{2\sqrt{6}}$$

$$y = 3 \cos(4\sqrt{6}t) - \frac{1}{2\sqrt{6}} \sin(4\sqrt{6}t) \leftarrow \text{Particular sol'n}$$

Graph

