Section 6.3 Question 7

Forced oscillations with damping has a form of $LQ'' + RQ' + \frac{1}{C} = E(t)$, $Q(0) = Q_0$, $Q'(0) = I_0$.

6.3 RLC questions

7) Find the steady current in the circuit

$$\frac{1}{20}Q'' + 2Q' + 100Q = 10\cos(2t) - 5\sin(2t)$$

In other words the question is to find Q_p' or Y_p' because $Q'(0) = I_0$ and it is translated to $Q_p' = I_p$ which is steady current.

$$r = \frac{-2 \pm \sqrt{2^2 - 4(100)(\frac{1}{20})}}{2(\frac{1}{20})} \quad r = \frac{-2 \pm \sqrt{-16}}{\frac{1}{10}} \quad r_1 = \frac{-2+4i}{\frac{1}{10}} \quad r_2 = \frac{-2-4i}{\frac{1}{10}}$$

$$r_1 = -20 + 40i \quad r_2 = -20 - 40i$$

$$\lambda = -20 \quad \omega = 40$$

$$y_1 = e^{-20t}\sin(40t)$$

$$y_2 = e^{-20t}\cos(40t)$$

undetermined coefficients:

$$y = B\sin(2t) + A\cos(2t)$$

$$y' = 2B\cos(2t) - 2A\sin(2t)$$

$$y'' = -4B\sin(2t) - 4A\cos(2t)$$

simplify main equation

$$(20)\ \frac{1}{20}y'' + 2y' + 100y = 10\cos(2t) - 5\sin(2t)$$

$$y'' + 40y' + 2000y = 200\cos(2t) - 100\sin(2t)$$

$$-4B\sin(2t) - 4A\cos(2t) + 1000B\cos(2t) - 1000A\sin(2t) + 2000B\sin(2t) + 2000A\cos(2t) = 200\cos(2t) - 400\sin(2t)$$

$$1375B\sin(2t) - 1000A\sin(2t) + 1000B\cos(2t) + 1375A\cos(2t) = 200\cos(2t) - 200\sin(2t)$$

$$\begin{cases} 1375B - 1000A = -100 \\ 1000B + 1375A = 200 \end{cases}$$

$$1375B = -100 + 1000A \rightarrow B = \frac{-100}{1375} + \frac{1000A}{1375}$$

$$B = -\frac{4}{55} + \frac{8}{11}A$$

$$1000\left(-\frac{4}{55} + \frac{8}{11}A\right) + 1375A = 200$$

$$-\frac{800}{11} + \frac{8000}{11}A + 1375A = 200$$

$$\boxed{A = \frac{24}{185}}$$

$$B = \frac{4}{55} + \frac{8}{11}\left(\frac{24}{185}\right)$$

$$\boxed{B = \frac{4}{185}}$$

$$y_p = \frac{4}{185}\sin(2t) + \frac{24}{185}\cos(2t)$$

$$y_p' = I_p = \left(\frac{d}{dt}\right)\frac{4}{185}\sin(2t) + \frac{24}{185}\cos(2t)$$

$$I_p = \frac{4(2)}{185}\cos(2t) - \frac{24(2)}{185}\sin(2t)$$

$$\boxed{I_p = \frac{20}{37}\cos(2t) - \frac{120}{37}\sin(2t)}$$

$$I_p = \frac{20}{37}\left(\cos(2t) - 6\sin(2t)\right)$$ steady state current