In Exercise 9 find the steady state current in the circuit described by the equation.

9. $\frac{1}{10} Q'' + 6Q' + 250Q = 10\cos 100t + 30\sin 100t$

$$\frac{r^2}{10} + 6r + 250 = 0$$

$$r^2 + 60r + 2{,}500 = 0$$

$$(r+30)^2 + 1{,}600 = 0$$

$$r = -30 \pm 40i$$

$$Q_c(t) = e^{-30t}(c_1 \cos 40t + c_2 \sin 40t)$$

$$Q_p(t) = (a\cos 100t + b\sin 100t)$$

$$Q'_p(t) = 100(-a\sin 100t + b\cos 100t)$$

$$Q''_p(t) = -10{,}000(a\cos 100t + b\sin 100t)$$

$-1{,}000(a\cos 100t + b\sin 100t) + 600(-a\sin 100t + b\cos 100t)$

$+ 250(a\cos 100t + b\sin 100t) = 10\cos 100t + 30\sin 100t$

$-750a + 600b = 10$

$-600a - 750b = 30$

Solve for a in $-750a + 600b = 10$

Subtract $600b$ from both sides of the equation

$-750a = 10 - 600b$

$-600a - 750b = 30$

Divide each term in

$-750a = 10 - 600b$ by -750 and simplify

$$a = -\frac{1}{75} + \frac{4b}{5}$$

$$-600a - 750b = 30$$

Replace all occurrences of a in

$$-600a - 750b = 30 \text{ with } -\frac{1}{75} + \frac{4b}{5}$$

$$-600\left(-\frac{1}{75} + \frac{4b}{5}\right) - 750b = 30$$

Simplify the left side

$$8 - 1230b = 30$$

Solve for b in $8 - 1230b = 30$

Move all term not containing b to the right side of the equation.

$$-1230b = 22$$

Divide each term in $-1230b = 22$ by -1230 and simplify

$$b = -\frac{11}{615}$$

Replace all occurrences of b with $-\frac{11}{615}$ in each equation

$$a = -\frac{1}{75} + \frac{46}{5} \quad \text{with} \quad -\frac{11}{615}$$

$$a = -\frac{1}{75} + \frac{4\left(-\frac{11}{615}\right)}{5}$$

$$a = -\frac{17}{615}$$

$$a = -\frac{17}{615} \quad , \quad b = -\frac{11}{615}$$

Particular Solution

$$Q_p(t) = -\frac{1}{615}\left(17\cos 100t + 11\sin 100t\right)$$

To find the steady state current we have to find the derivative of particular solution

$$Q_p(t) = -\frac{1}{615}\left(17\cos 100t + 11\sin 100t\right).$$

Steady State Current

$$Q_p(t) = \frac{1}{615}\left(1{,}700\sin 100t - 1{,}100\cos 100t\right)$$

$$= \frac{340}{123}\sin 100t - \frac{220}{123}\cos 100t$$