Project #2

Group #1: Springs Problem 1
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Exercise 6.1: Question #11

11. A unit mass hangs in equilibrium from a spring with constant $k = 1/16$. Starting at $t = 0$, a force $F(t) = 3\sin t$ is applied to the mass. Find its displacement for $t > 0$.

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6.1 Spring Problem 1
Question #11

Differential Equation of Spring-mass undamped system:

$my'' + ky = F(t)$ Given: $m = 1$, $k = \frac{1}{16}$, $F(t) = 3\sin(t)$

$u(t) \to$ displacement at time t

$y'' + \frac{1}{16}y = 3\sin(t)$

$16y'' + y = 48\sin(t) \to 1$

Rough Work:

$16\cdot\left(y'' + \frac{1}{16}y\right) = (3\sin(t))\cdot 16$

$= 16y'' + y = 48\sin(t)$

$16m^2 \cancel{+1} = 0$
$\quad\ \ -1 \quad -1$

$\frac{\cancel{16}m^2}{\cancel{16}} = \frac{-1}{16}$

$\sqrt{m^2} = \sqrt{\frac{-1}{16}}$ $\quad m = \sqrt{\frac{1}{16}}\,i$

$m = \pm\frac{1}{4}i$

Auxiliary equation $16m^2 + 1 = 0$

$m = \pm\frac{1}{4}i$

$y_c = C_1\cos\left(\frac{t}{4}\right) + C_2\sin\left(\frac{t}{4}\right)$

Let particular solution $y_p = A\cos(t) + B\sin(t)$

$y_p' = -A\sin(t) + B\cos(t)$

$y_p'' = -A\cos(t) - B\sin(t)$

$\frac{d}{dt}\left(A\cos(t) + B\sin(t)\right)$

$= A\frac{d}{dt}(\cos(t)) + B\frac{d}{dt}(\sin(t))$

$y_p' = -A\sin(t) + B\cos(t)$

$\frac{d}{dt}\left(-A\sin(t) + B\cos(t)\right) = -A\frac{d}{dt}(\sin(t)) + B\frac{d}{dt}(\cos(t))$

$y_p'' = -A\cos(t) - B\sin(t)$

$16(-A\cos(t) - B\sin(t)) + A\cos(t) + B\sin(t) = 48\sin(t)$

$\cos(t)(-16A + A) + \sin(t)(-16B + B) = 48\sin(t)$

$\frac{-\cancel{15}B}{-\cancel{15}} = \frac{48}{-15}$ $\quad B = \frac{48}{-15}$

$y_p = -\frac{48}{15}\sin(t) \to y_p = -\frac{16}{15}\sin(t)$

$y = C_1\cos\left(\frac{t}{4}\right) + C_2\sin\left(\frac{t}{4}\right) - \frac{16}{15}\sin(t) \to y_c + y_p$

$y(0) = 0 \to C_1 = 0$

$y'(t) = -\frac{1}{4}C_1\sin\left(\frac{t}{4}\right) + \frac{1}{4}C_2\cos\left(\frac{t}{4}\right) - \frac{16}{15}(\cos(t))$

$y'(0) = 0 \to 0 = C_2\cdot\frac{1}{4} - \frac{16}{15} \to C_2 = \frac{64}{15}$

displacement is $\frac{64}{15}\sin\left(\frac{t}{4}\right) - \frac{16}{15}\sin\left(\frac{t}{4}\right)$ $\quad \boxed{y(t) = \frac{16}{15}\left(4\sin\left(\frac{t}{4}\right) - \sin(t)\right)}$