Problem 3 (Test 1) Version C

Find the general solution for the following differential equation: $y' - 5y = \frac{1}{y^2}$

This is a Bernoulli eqn because it is of the form

$y' + P(x)y = f(x)\,y^r$ $\quad$ i.e. $(P(x) = -5,\ r = -2,\ r-1 = -3$, $f(x) = 1)$

we solve this using this 2 steps:

(i) Associated homogeneous (ii) we solve for $y = uy_1$

(i) associated homog: $-\int P(x)\,dx = -\int -5\,dx$

$= 5x + C$

Ce^{5x+C} then $y_1 = e^{5x}$

(ii) $y = u \cdot y_1$

$\frac{u'}{u^r} = f(x)\,y_1^{r-1}$

$\frac{u'}{u^2} = 1 \cdot e^{5x(-3)}$

$u^2 \cdot u' = e^{-15x}$

integrate both sides wrt x

$\int u^2 \frac{du}{dx}\,dx = \int e^{-15x}\,dx$

$\frac{u^3}{3} = \frac{-e^{-15x}}{15} + C$

$u^3 = \frac{-e^{-15x}}{5} + C$

$u = \sqrt[3]{\frac{-e^{-15x}}{5} + C} \cdot e^{5x}$