Ken Mei
Test #1 Solutions:
Version A
Question #3) Find the general solution to the differential equation: $xy^1 = xy + y^2$

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Version A
Question 3:
Find the general solution for the following differential equation:

$$x^2 y' = xy + y^2 \qquad \frac{y^2}{x^2} = \left(\frac{y}{x}\right)^2$$

$$\frac{\cancel{x^2} y'}{\cancel{x^2}} = \frac{xy}{x^2} + \frac{y^2}{x^2} \rightarrow y' = \frac{y}{x} + \left(\frac{y}{x}\right)^2$$

Let $u = \frac{y}{x}$

$g(u) = u + u^2$, Step 1: $y_1 = x$, Step 2: $\frac{u'}{g(u)-u} = \frac{1}{x}$, Step 3: $y = u y_1$

Step 2:

$$\frac{u'}{\cancel{u} + u^2 - \cancel{u}} = \frac{1}{x} \rightarrow \frac{u'}{u^2} = \frac{1}{x} \rightarrow \int \frac{u'}{u^2}\, dx = \int \frac{1}{x}\, dx$$

$$\int \frac{1}{u^2}\, du = \int \frac{1}{x}\, dx \rightarrow \int u^{-2}\, du = \int \frac{1}{x}\, dx$$

$$\frac{u^{-1}}{-1} = \ln|x| + C = \frac{-1}{u} = \ln|x| + C$$

$$\cancel{u} \cdot \frac{-1}{\cancel{u}} = (\ln|x| + C) \cdot u \rightarrow \frac{-1}{\ln|x| + C} = \frac{\cancel{(\ln|x| + C)} \cdot u}{\cancel{\ln|x| + C}}$$

$u = \frac{-1}{\ln|x| + C}$ Step 3: $y = u \cdot y_1$, $y_1 = x$

$$y(x) = \left(\frac{-1}{\ln|x| + C}\right) \cdot x = \boxed{y(x) = \frac{-x}{\ln|x| + C}}$$

general solution