Cheat sheet_MAT2680_Test#1.

Linear Homogeneous:

$$y' + p(x)y = 0$$

General solution:

$$y = Ce^{-\int p(x)\,dx}$$

Linear Non-homogeneous:

$$y' + p(x)y = f(x)$$

* $u = \int \frac{f(x)}{y_1}\,dx$
* Use Variation of parameters
* y_1 is any solution associated with homogeneous eq.

General solution:

$$y = u y_1.$$

Separable Equations:

$$h(y)y' = g(x)$$

To Solve:

$$\int h(y)\frac{dy}{dx}dx = \int g(x)\,dx$$

$$\int h(y)\,dy = \int g(x)\,dx$$

(Evaluate & Solve for y)

Bernoulli Eq.

$$y' + p(x)y_1 = f(x)\,y^r$$

To Solve:

* y_1 is any solution associated with homogeneous Eq.
* Variation of Parameters:
  Set $y = uy_1$ and $y' = u'y_1 + uy_1'$
  plug into $y' + p(x)y_1 = f(x)y^r$.
* Evaluate until $\frac{u'}{u^r} = f(x)y_1^{r-1}$

Non-Linear Homogeneous Eq.

$$y' = f(x,y) \quad \Big\} \; y = uy_1 = u\cdot x.$$

To Solve: a) Find $q(u)$ by subs. $u = \frac{y}{x}$. Then $\frac{u'}{q(u)-u} = \frac{1}{x}$,

Q. Does it depend on $\frac{y}{x}$? The integrate & Solve for

$$y = u\cdot x.$$

Exact Equations:

① Find $M(x,y)$ and take ~~Deriva~~ $\frac{dy}{dx}$ to get M_y.

② Find $N(x,y)$ and take $\frac{d}{dx}$ to get N_x.

Q. Does $M_y = N_x$? If so it is Exact.

Then, $F(x,y) = \int M(x,y)\,dx$

$\int M(x,y)\,dx + \varnothing y$.

Then, $\frac{dy}{dx}\left[\int M(x,y)\,dx + \varnothing' y\right] = F_y(x,y)$.

$F_y(x,y) = N(x,y)$ * to get value of $\varnothing' y = ?$

Then, $\int \varnothing' y = \varnothing y$

Finally, $F(x,y) + \varnothing y = C$ ← General Solution.

Table of Integrals:

$\int \sin u\,du = -\cos u + C$ $\qquad$ $\int \ln u\,du = u \ln u - u + C$

$\int \cos u\,du = \sin u + C$ $\qquad$ $\int \sec^2 u\,du = \tan u + C$

$\int \tan u\,du = \ln|\sec u| + C$ $\qquad$ $\int e^u\,du = e^u + C$

$\int \sec u \tan u\,du = \sec u + C$ $\qquad$ $\int \frac{1}{u \ln u}\,du = \ln|\ln u| + C$