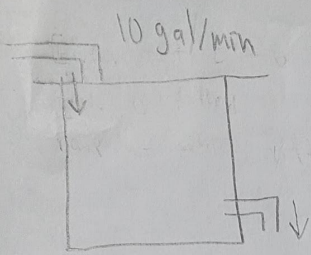Group 1
Section: 4.1 Growth and Decay
Group Members: Ken Mei and Luijen Payano
4.1 Question #17:

17. A tank is empty at $t = 0$. Water is added to the tank at the rate of 10 gal/min, but it leaks out at a rate (in gallons per minute) equal to the number of gallons in the tank. What is the smallest capacity the tank can have if this process is to continue forever?

10 gal/min   4.1 Question #17

Let $x(t)$ be the amount of water at any time t,

$$x'(t) = 10 - x(t)$$

$$x'(t) + x(t) = 10$$

$$If = e^{\int dt} = e^t$$

$$e^t x'(t) + e^t x(t) = e^t \cdot 10$$

$$\frac{d}{dt}\left[e^t x(t) \right] = 10 e^t$$

$$e^t x(t) = 10 e^t + C \longrightarrow x(t) = \frac{10 e^t}{e^t} + \frac{C}{e^t}$$

$$x(t) = 10 + Ce^{-t} \qquad = 10 + Ce^{-t}$$

at $t = 0$, $x(t) = 0$

$$0 = 10 + Ce^{-(0)}$$

$$(e^{-(0)} = (\quad \frac{10 + Ce^{-0} = 0}{-10 \qquad -10}$$

$$\frac{}{C = -10}$$

$\longrightarrow$ $x(t) = 10 + Ce^{-t}$

Solved for $C = -10$

$$x(t) = 10 - 10e^{-t}$$

$$= 10(1 - e^{-t})$$

$$x(t) = 10(1 - e^{-t})$$

If $t \to \infty$, $x(t) \to 10$

Smallest capacity of tank is 10 gallons