Project 4 Predator-Prey Model

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Differential equations are extremely important when it comes to real world applications. They are used to model many different phenomena in fields such as engineering, physics, economics, etc. Their main use is to predict the behavior of complex systems. They can predict the movement of celestial objects, dynamics of fluid, heat, and electricity, spread of epidemics, finances and many more. The application that we chose for this project is the Predator-Prey model. A predator-prey model is used to describe the interaction between two species in any given ecosystem, where one species feeds on the other. In our case the predator and the prey. For example, a fox and a rabbit, a bear and a fish or even a cat and a mouse. The predator-prey model typically uses a system of two first order differential equations to describe the dynamics of the two species. These equations are often called the Lotka-Volterra equations, and they take into account factors such as the rate at which predators catch and eat prey, and the rate at which prey reproduce. The Lotka-Volterra equations vary depending on the specific interaction that you are trying to model, but for the most part they look like this:

$$\frac{dx}{dt}=αx-βxy$$

$$\frac{dy}{dt}=-γy+δxy$$

Where x is the number of prey, y is the number of predators and α, β, γ, δ are parameters.

We can dissect each equation into two sections to understand how it works. Let’s take the first equation. $$\frac{dx}{dt}=αx-βxy$$

Here, *αx* represents the prey population, which by itself is positive and will continue to grow exponentially since there is no predator to stop it. That’s where -*βxy* comes in. This section shows us how the interaction between prey x and predator y work. The reason why it is negative is because if some prey encounters a predator, the predator will come on top, therefore it leads to a prey population decrease.

 The second equation uses the same theory but backwards.

$$\frac{dy}{dt}=-γy+δxy$$

Here, *-γy* represents the predator population. The reason that it is negative is because a predator by itself will have no prey to eat, so their population is bound to decrease. Similarly to the first equation, *δxy* indicates the interaction between prey x and predator y. In this equation this section is positive because if a predator encounters prey, the predator will come on top, resulting in a population “increase” for the predator.

 Finally, solving these equations can yield 3 types of results. If both are 0, (0, 0) then that means both populations are 0. If one of the equations yields a number, [ (α, 0) or (0, b) ], that means one of the populations is stable while the other is not. If both yield a number, (a, b), then both populations are stable. Below is a link to make a model by yourself, by inputting the parameters.

Make your own model:

<https://demonstrations.wolfram.com/PredatorPreyModel/>

Sources:

<http://calculuslab.deltacollege.edu/ODE/7-B-1/7-B-1-h.html>

<https://youtu.be/Zg9k9ijiYPA>

<https://www.kristakingmath.com/blog/predator-prey-systems>

<http://www.scholarpedia.org/article/Predator-prey_model>