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Population Growth

To be able to find out a differential equation that can tackle the problem of population growth. The Malthusian Growth Model is what will be used for this problem. This was developed in 1798 by a British economist and demographer Thomas Robert Malthus. He had written an essay called “An Essay on the Principle of Population” which described his arguments of population of growth. He mentions that population growth will always outrun growth of production if left unchecked. This paper contained his mathematical model that could calculate population growth. The model is able to depict exponential growth of a population and it uses differential equations to do so. The main idea of this model is to follow the assumption that the rate at which a population of a country grows at a certain time is proportional to the total population of the country at that time. The formula goes as is:

(dP) / (dt) = kP(t)

P(t) = P0e^kt

* P(t) is the total population at time t and k is the growth constant or the decay constant.
* The parameters go as is k<0, then there is decay while if k>0, there is growth.
* When t = 0 or p(0), this is given for the initial population and still contains the parameters for k.

However, this model does not take into account external factors such as environmental limitations and others of this type. Due to the fact that population growth naturally does not take into account external resources or the level production needs to be to maintain the population growth as stated in his paper. There needs to be another model implemented to provide a solution. This gives way to the logistic model also known as the Verhulst-Pearl model. The differential equation for this model is:

dp / dt = kP (1-p/M)

* This equation accounts for the limiting factors of the environment which is M (also known as the carrying capacity).
* The equation shows that it can be separable. We can get the p to one side so that the k is left alone on the right.

dp / (P (1-p/M)) = kdt

* This allows to integrate and use partial fraction decomposition to solve.

P/(1-P/M) = Ce^kt

* We can then multiply the denominator on both sides and do some simplifying to get P alone.

P = (MCe^kt) / (M+Ce^kt)

Final Thoughts: While researching this project and looking for a topic I found this interesting because I always wondered how it is possible for us to have a good estimate as to how many humans are on earth. When reading the sites about this differential equation, I realized the little nuances that can slip by that actually have a big impact on the numbers. Like the environmental capabilities and other aspects like it. Initially, I did not really think of it cause environments around the world are very different so it is kind of difficult to account for each of them.

References

“Differential Equations.” Population Growth, <https://math24.net/population-growth.html>.

“Thomas Malthus.” Encyclopædia Britannica, Encyclopædia Britannica, Inc., <https://www.britannica.com/biography/Thomas-Malthus>.

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