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Project 4
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**Population growth with food supply**:

Around the world, over one in seven individuals experience chronic hunger and lack the nourishment they need to stay healthy and lead active lives. This is despite the fact that there is sufficient food for everyone in the globe. Producers are under pressure to supply an increased demand for food as overpopulation sets in, placing severe restrictions on production and delivery systems. The Food and Agriculture Organization projects that by 2050, population and economic growth will result in a doubling of demand for food globally. Differential equations can be used to represent the size of a population as it varies over time.

To model population growth using a differential equation, we first need to introduce some variables and relevant terms. To represent time, use the variable t. It is possible to measure time in hours, days, weeks, months, or even years. The units that are utilized in each problem must be specified. The variable $P$ will represent population. Since the population varies over time, it is understood to be a function of time. Therefore, we use the notation $P(t)$ for the population as a function of time. If $P(t)$ is a differentiable function, then the first derivative $d(P)/d(t)$ represents the instantaneous rate of change of the population as a function of time. The formula is given by:

$$d(P)/d(t) = rP(t)$$

And the exponential growth function is:

$$P(t) = Po e^{rt}$$

- *P(t)* represents the population at time *t*,
- *Po* represents the initial population (population at time *t*=0),
- the constant r >0 is called the growth rate and r < 0 is called decay rate

The rate of population growth is therefore proportional to the population at that moment, according to the differential equation. Furthermore, it asserts that the proportionality constant remains constant.

One problem with this function is its prediction that as time goes on, the population grows without bound. This is unrealistic in a real-world setting. The growth constant is a measure of the rate at which an organism's population grows over time and can be interpreted as a net (birth minus death) percent growth rate per unit time. Biologists have found that in many biological systems, the population grows until a certain steady-state population is reached Various factors limit the rate of growth of a particular population, including birth rate, death rate, food supply, predators, and so on. The concept of carrying capacity allows for the possibility that only a certain number of organisms or animals can thrive without running into resource issues. This where comes in play **the logistic differential equation.** Let k represent the carrying capacity for a particular organism in a given environment, and let r be a real number that represents the growth rate the Function P(t) represents the population of this organism as a Function of time t, and the constant $P0$ represents the initial population (population of the organism at time t=0). Then the *logistic differential equation* is:

$$d(P)/d(t) = rP\ (1-P/K)$$

This logistic differential equation describes the situation where a population grows proportionally to its size, but stops growing when it reaches the size of K.

As Thomas Robert Malthus states: “Population increases in a geometric progression while food production increased in arithmetic progression. Thus, population grew faster than food production and tend to outstrip it in a short time. Unless humans can limit reproduction voluntarily through self-restraint, population would be reduced by catastrophic events such as diseases, starvation, misery, and wars”. Differential Equation is very helpful in keeping track of the population growth to manage food supply.

**Reference:**

**LAND RESOURCE EXPLOITATION: FOOD PRODUCTION**
**https://web.ccsu.edu/faculty/kyem/GEOG473/5thWeek/Food%20production%20and%20GM%20foods.htm**
**Why Population Matters to Food Security**
https://toolkits.knowledgesuccess.org/toolkits/family-planning-advocacy/why-population-matters-food-security
**The Logistic Equation**
https://math.libretexts.org/Bookshelves/Calculus/Book%3A_Calculus_(OpenStax)/08%3A_Introduction_to_Differential_Equations/8.04%3A_The_Logistic_Equation