No.

Date

Imerson Memko MAT 2680 Practice Exam

1) $ty' + 5y = 9t - 6t^2 - 2$ $\qquad p(x) = \frac{5}{t}$

$$y' + \left(\frac{5}{t}\right)y = 9 - 6t - \frac{2}{t} \qquad f(x) = 9 - 6t - \frac{2}{t}$$

$$y_1 = e^{-\int \frac{5}{t} dt} \Rightarrow y_1 = t^{-5}$$

$$u = \int \frac{f(x)}{y_1} dt$$

$$u = \int \frac{9 - 6t - \frac{2}{t}}{t^{-5}}$$

$$u = \int \left(9 - 6t - \frac{2}{t}\right) t^5 dt$$

$$u = \frac{9t^6}{6} - \frac{6t^7}{7} - \frac{2t^5}{5} + c$$

$$y = \left(\frac{3}{2}t^6 - \frac{6}{7}t^7 - \frac{2}{5}t^5 + c\right)t^{-5}$$

$$\boxed{y = \frac{3}{2}t - \frac{6}{7}t^2 - \frac{2}{5} + \frac{c}{t^5}}$$

No

Date

2) $-(18x+8y)+(16y-8x)\,y'=0$

$M_y = (-18x-8y)\frac{d}{dy}$ $= -8$

$N_x = (16y-8x)\frac{d}{dx}$ $= -8$

Exact Equation

$F_x(x,y) = -18x-8y$

$F_y(x,y) = 16y-8x$

$f = \int -18x-8y \, dx$

$= -18\frac{x^2}{2} - 8yx$

$= -9x^2-8yx+\phi(y)$

$f_y = \frac{d}{dy}\left(-9x^2-8yx+\phi(y)\right)$

$= 0-8x+\phi'(y)$

$-8x+\phi'(y) = 16y-8x$

$\phi'(y) = 16y$

$\phi(y) = \int 16y \, dy$

$\phi(y) = 16\frac{y^2}{2} = 8y^2$

$\boxed{-9x^2-8yx+8y^2=c}$

$y(-6)=7 \Rightarrow c = -9(-6)^2-8(7)(-6)+8(7)^2 = \boxed{404}$

Date

3) $t^2y' - 2ty = 9y^2$

$$y' - \left(\frac{2}{t}\right) y = \left(\frac{9}{t^2}\right) y^2$$

$$y_1 = e^{-\int -\frac{2}{t}\,dt}$$

$$y_1 = t^2$$

$$\frac{v'}{v^r} = f(x)\, y_1^{\,r-1}$$

$y = v y_1$

$p(x) = -\frac{2}{t}$

$f(x) = \frac{9}{t^2}$

$r = 2$

$$\frac{v'}{v^2} = \left(\frac{9}{t^2}\right)(t^2)^{2-1}$$

$$\frac{v'}{v^2} = 9$$

$$\int \frac{1}{v^2}\,dv = \int 9\,dt$$

$$-\frac{1}{v} = 9t + c$$

$$v = -\frac{1}{9t+c}$$

$$y = \left(-\frac{1}{9t+c}\right)t^2$$

$$\boxed{y = -\frac{t^2}{9t+c}}$$

No

Date

4) $y'' + 16y' + 64y = 108e^{-2t}$

$y'' + 16y' + 64y = 0$

$r^2 + 16r + 64 = 0$

$(r+8)(r+8)$ $\quad r_1 = r_2 = -8$

$y_1 = e^{-8t}$, $y_2 = te^{-8t}$

$Y_h = Ae^{-8t} + Bte^{-8t}$

$Y_p = Ae^{-2t}$

$Y_p' = A(-2e^{-2t})$

$Y_p'' = A(4e^{-2t})$

$A(4e^{-2t}) + 16(A(-2e^{-2t})) + 64Ae^{-2t} = 108e^{-2t}$

$4Ae^{-2t} - 32Ae^{-2t} + 64Ae^{-2t} = 108e^{-2t}$

$4A - 32A + 64A = 108$

$A = 3$ $\quad Y_p = 3e^{-2t}$

$$\boxed{y = Ae^{-8t} + Bte^{-8t} + 3e^{-2t}}$$

5) $x^2y'' - 7xy' + 16y = 0 \qquad y_1 = x^4$

$y = ux^4$

$y' = u'x^4 + 4ux^3$

$y'' = u''x^4 + 4u'x^3 + 4u'x^3 + 12ux^2$

$= u''x^4 + 8u'x^3 + 12ux^2$

$x^2(u''x^4 + 8u'x^3 + 12ux^2) - 7x(u'x^4 + 4ux^3) + 16ux^4$

$= u''x^6 + 8u'x^5 + 12ux^4 - 7u'x^5 - 28ux^4 + 16ux^4$

$= u''x^6 + u'x^5$

$\Downarrow$

$x^5(u''x + u') = 0$

$u''x + u' = 0 \qquad z = u'$

$z'x + z = 0$

$z'x = -z$

$z' = -\frac{z}{x}$

$\frac{dz}{dx} = -\frac{z}{x}$

$\frac{1}{z}\,dz = -\frac{1}{x}\,dx$

$\ln(z) = -\ln(x) + c_1$

$e^{\ln|z|} = e^{-\ln|x|}$

$z = \frac{1}{x} \Rightarrow u' = \frac{1}{x}$

$u = \int \frac{1}{x}\,dx = \ln|x| + c$

$y = (\ln|x| + c)x^4$

$\boxed{y = x^4 \ln|x| + cx^4}$