Black-Scholes Equation

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-rT}$$

$$d_1 = \frac{ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$C(S,t)$	(call option price)
$N(\ )$	(cumulative distribution function)
$T = (T_1 - t)$	(time left til maturity (in years))
S	(stock price)
K	(strike price)
r	(risk free rate)
σ	(volatility)

A mathematical method, developed in 1973 by Fischer Black, Robert Merton and Myron Scholes, is one of the most important concepts in modern financial theory. This is commonly used when estimating the theoretical value of derivatives based on other investment instruments, with respect to the impact of time and other risk factors(call option price). To put it simply, it is a differential equation used to determine a fair price for option contracts. Even further into detail, an option contract is an agreement put in place, between two parties which facilitates a future transaction that involves an asset at a preset price and date.

Now to understand how the Black-Scholes equation/model works, we must first understand what makes up the equation. There are five requirements which are time until the expiration of the option, stock price(underlying asset price), the strike price of the option, the risk free interest rate and the volatility. The first requirement explains itself, the stock price otherwise known as the price of the underlying asset, means the current price that a share of stock is trading for on the market. The strike price essentially is a set price, where the owner of the option contract can buy/sell some underlying security(stock/bond) at a pre-specified price. Risk free rate refers to the rate of return of an investment with no risk. And lastly, volatility, which is a statistical measurement of the rate for which a stock either increases or decreases. Once these variables are gathered, the rational prices for options being sold, can be theoretically assumed.

The Black-Scholes equation is widely used, but take it with a grain of salt. For the theoretical value to be estimated, a lot of assumptions must be taken into consideration. Excess cash(dividends) are not paid out during the life of the option. Markets are not predictable, which makes market movements random. When buying the option there are no transaction costs. Both the risk-free rate and volatility should be known and constant. The returns of the underlying asset are normally distributed. The option is European, which is a version of an options contract that limits certain executions to the expiry date. This means should a stock move in price, the investor cannot sell the shares. Now, in reality there is a margin of error for several of these assumptions. An example would be that the Black-Scholes model assumes that volatility remains constant throughout the option's life, which is incorrect. That is because it is always fluctuating with the level of supply and the demand. It is also the toughest input to determine.

A real world example of this would be, let's say we are interested in estimating a value for a call option(a contract to buy a stock until an expiration date) from Tesla. Using Yahoo finance today, we find that Tesla's stock price(S) is $245. Its exercise price, other words known as strike price(K), is $294. From yahoo we find that tesla has a 20% higher strike price than the stock price. After googling we find that the time to expiration(T) is 101 days. Now we'll assume that the risk-free interest rate(r) is 2.12% (based on the US 10-year government bonds). The volatility can be estimated by observing its historical prices, which is 0.38%. Now that all five requirements have been gathered, we substitute into the formula and solve. To make things simpler, and is used commonly today in the stock market, an online Black-Scholes Equation calculator. After putting in the input we find that the call option is priced around $6.

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