Fluid Dynamics

Problems with fluid flows may be found every day in industrial operations, meteorology, and the design of aircraft and engines. You can determine the forces acting on an airplane by applying applications of fluid dynamics. The ability of designers to improve aerodynamic properties will be limited without computational fluid dynamics. Solving fluid dynamics issues is one highly practical use of differential equations.

To address fluid flow challenges including velocity, density, and chemical compositions, computational fluid dynamics uses data structures. Partial differential equations that describe the laws of conservation of mass, momentum, and energy are used to quantify the flow behavior of gasses and liquids.

You utilize the Navier Stokes equation to address issues in fluid dynamics. The Euler Equation may be produced from the equations below by using viscous processes.

Navier–Stokes Equations

3 – dimensional – unsteady

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Coordinates: (x,y,z)

Velocity Components: (u,v,w)

Time : t

Density: ρ

Total Energy: Et

Pressure: p

Stress: τ

Heat Flux: q

Reynolds Number: Re

Prandtl Number: Pr

Continuity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

X – Momentum:

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right]$$

Y – Momentum:

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right]$$

Z – Momentum

$$\frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho uw)}{\partial x} + \frac{\partial(\rho vw)}{\partial y} + \frac{\partial(\rho w^2)}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right]$$

Energy:

$$\frac{\partial(E_T)}{\partial t} + \frac{\partial(uE_T)}{\partial x} + \frac{\partial(vE_T)}{\partial y} + \frac{\partial(wE_T)}{\partial z} = -\frac{\partial(up)}{\partial x} - \frac{\partial(vp)}{\partial y} - \frac{\partial(wp)}{\partial z} - \frac{1}{Re_r Pr_r}\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right]$$
$$+ \frac{1}{Re_r}\left[\frac{\partial}{\partial x}(u\,\tau_{xx} + v\,\tau_{xy} + w\,\tau_{xz}) + \frac{\partial}{\partial y}(u\,\tau_{xy} + v\,\tau_{yy} + w\,\tau_{yz}) + \frac{\partial}{\partial z}(u\,\tau_{xz} + v\,\tau_{yz} + w\,\tau_{zz})\right]$$

Here partial derivatives are represented by the letter d: ∂

The equations, which consist of a series of linked differential equations, might theoretically be resolved for a specific flow issue using calculus-based techniques. However, in reality, these equations are too complex to be solved analytically. Engineers in the past continued to approximate and simplify the equation set until they obtained a set of equations that they could solve. Recently, approximations to the equations utilizing a number of approaches, including as finite difference, finite volume, finite element, and spectral methods, have been solved using high speed computers. Computational fluid dynamics, sometimes known as CFD, is this field of research.

We may also use Bernoulli's equation to assist us solve fluid pressure problems in practical situations. Height, velocity, cross-sectional area, density, and other variables may be included in these equations.

I thought the fluid moving through a pipe with variable cross-sectional area and height was an intriguing example. It was claimed that if given the density, pressure, and velocity of the fluid entering one side of the pipe, as well as the cross sectional area of both apertures, we would need to determine the two unknowns (velocity and pressure) of a fluid departing the pipe.

Fire hoses with an internal diameter of 6.40 cm are utilized in large building fires. Let's say that such a hose has a flow rate of 40 L/s and a gauge pressure of $1.62 \times 10^6 N/m^2$. The hose ascends a ladder 10 meters before arriving at a nozzle with an inner diameter of 3 cm.

What is the pressure inside the nozzle, assuming there is little resistance?

Since depth is not constant, we must apply Bernoulli's equation to find the pressure in this situation.

According to Bernoulli's equation:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2,$$

the beginning conditions at ground level and the end circumstances inside the nozzle, respectively, are denoted by the subscripts 1 and 2, respectively. First, we need to determine the speeds V_1 and V_2.

Q = $A_1 V_1$, therefore we have:

$$v_1 = \frac{Q}{A_1} = \frac{40\ \times\ 10^{-3}\ \mathrm{m^3/s}}{\pi(3.20\ \times\ 10^{-2}\ \mathrm{m})^2} = 12.4\ \mathrm{m/s}.$$

Likewise, we discover:

$$v_2 = 56.6\ \mathrm{m/s}.$$

Reaching the fire is made easier by this relatively high speed. We now resolve Bernoulli's equation for P_2 by setting h_1 to zero:

$$P_2 = P_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) - \rho g h_2.$$

The results of replacing known values gives us:

$$P_2 = 1.62 \times 10^6\ \mathrm{N/m^2} + \frac{1}{2}(1{,}000\ \mathrm{kg/m^3})\left[(12.4\ \mathrm{m/s})^2 - (56.6\ \mathrm{m/s})^2\right] - (1{,}000\ \mathrm{kg/m^3})(9.80\ \mathrm{m/s^2})(10.0\ \mathrm{m}) = 0.$$

Now that Bernoulli's equation has been applied, we can get the pressure leaving the nozzle.

Reference

https://www.grc.nasa.gov/www/k-12/airplane/nseqs.html

https://www.texasgateway.org/resource/123-most-general-applications-bernoullis-equation