2.3 Separable Equations

$y' = \cos^2(5x)\cos^2(3y)$ separate then integrate for general solution

$\downarrow$

$$\frac{dy}{\cos^2 3y} = \cos^2 5x \rightarrow \int \frac{1}{\cos^2 3y}\,dy = \int \cos^2 5x\,dx$$

$\downarrow$

$$\text{gen sol} = \frac{\tan 3y}{3} = \frac{\sin 10x + 10x}{20} + c$$

D2 Inverse Laplace Transform

$$f(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2} + \frac{7}{s+9}\right\} = t + 7e^{-9t}$$

Laplace Transforms

$* \; t^n \leftrightarrow \frac{n!}{s^{n+1}}$ $\qquad * \; e^{at} \longleftrightarrow \frac{1}{s-a}$

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Final Review

6.4 2nd Order Repeated

$y'' + 10y' + 25 = 0 \rightarrow r^2 + 10r + 25 \quad (r+5)(r+5) \quad r = -5$

a) general solution $= Ae^{-5t} + Bte^{-5t}$

b) particular solution when $y(0) = 1$ and $y'(0) = -10$

y: $1 = A \cdot e^0 + 0 \rightarrow 1 = A$

y': $-5Ae^{-5t} + Be^{-5t} - 5Bte^{-5t}$

sub $-5 \cdot 1 + B - 0 = -10 \rightarrow -5 + B = -10 \rightarrow B = -5$

$\underline{+5} \quad \underline{+5}$

particular solution $= e^{-5t} - 5te^{-5t}$

7.2 2nd Order Complex

$y'' - 14y' + 130y = 0$

a) characteristic polynomial $= r^2 - 14r + 130$

b) roots

$r^2 - 14r + 130 \rightarrow \dfrac{14 \pm \sqrt{-14^2 - 4 \cdot 1 \cdot 130}}{2 \cdot 1} = r = 7 \pm 9i \qquad \lambda = 7 \quad \omega = 9$

c) basic solutions

$y_1(t) = e^{7t} \cos 9t \qquad y_2(t) = e^{7t} \sin 9t$

1.4 1st Order Linear

$y' + 3y = 4t + 3e^{-2t} \qquad p(t) = 3 \quad f(t) = 4t + 3e^{-2t} \qquad y = ce^{-3t} \quad y_1 = e^{-3t}$

integration by parts
$\int fg' = fg - \int f'g$

$u = \int \dfrac{4t + 3e^{-2t}}{e^{-3t}}\, dt \rightarrow \int e^{3t}(4t + 3e^{-2t})\, dt \qquad f = 3e^{-2t} + 4t \quad g' = e^{3t} \qquad f' = -6e^{-2t} + 4 \quad g = \dfrac{e^{3t}}{3}$

$\dfrac{(3e^{-2t} + 4t)e^{3t}}{3} - \int \dfrac{(-6e^{-2t} + 4)e^{3t}}{3}$

common denominator to enable subtraction

$\int \dfrac{6e^{t} + 4e^{3t}}{3} = \int -2e^{t} + \int \frac{4}{3}e^{3t} = -2\int e^{t} + \frac{4}{3}\int e^{3t} = -2e^{t} + \dfrac{4e^{3t}}{9}$

$\dfrac{(3e^{-2t} + 4t)e^{3t}}{3} + 2e^{t} - \dfrac{4e^{3t}}{9} + C = \dfrac{(12t - 4)e^{3t} + 27e^{t}}{9} + C$

$y = u \cdot y_1 = e^{-3t}\left(\dfrac{(12t-4)e^{3t} + 27e^{t}}{9} + C\right)$