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MAT 2680

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**Three-Body Problem**

Differential equation is used throughout many fields, for example physics. In physics, one of the biggest problem was the three-body problem. The three-body problem, according to Wikipedia, is the problem of taking the initial positions and velocities of three point masses and solving for their future motion. The laws connected to three-body problem are Newton's laws of motion and Newton's law of universal gravitation. In a lecture by Juhan Frank, it’s stated that the earliest appearance of the three-body problem is after Newton solved the two-body problem, which proved Moon orbited around the Earth. For the general three-body problem, it is useful to work in the center-of-mass (CM) system with xi denoting the position of mass Mi. The differential equation to use is,

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In this equation, j, k, and l represent 1, 2, and 3. Under the CM condition, its first derivative is,

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The first derivative reduces the order of the system from 18 to 12. Also, with the absence of external forces and torques, the energy and angular momentum are conserved quantities or integrals of the motion. This decrease the order of the system little more to 8. After being decrease to 4, the motion was restricted to a plane fixed in space. This became unsolvable. In 1973, two people, Broucek & Lass, realized that the equations of motion could be written in a more balance or symmetrical form by using the relative position vectors Si = Xj − Xk. So, you label the equation in a way that the Si is on the opposite of the vertex of the triangle containing the mass Mi.



Then, the relative position vectors, the equations of motion, took this form,

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Where M = m1 + m2 + m3 is the total mass and the vector G is,  
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While the Si equation part is similar to another known problem called the two-body Kepler problem, the vector G was the reason for the difficulty in this problem since it couples with the equations for Si. One of the solution they applied to the three body problem was Euler’s Solution, which is also connected to differential equations.

Using Euler’s solution, if all particles were collinear, the vectors Si , Xi and G are proportional to one another. For example, if M2 lied in between the other two masses, then S3 would point from M1 to M2, S1 would point in the same direction, and S2 would point back from m3 to m1. With this information, we can write

both equations of motion are in terms of S3 and lambda (λ), and λ is a positive scalar.

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The two-body equation above, as stated in the lecture, is a fifth degree polynomial in λ with one single positive real root which is a function of the three masses. Now, the particles move along confocal ellipses of the same eccentricity and the same orbital period around the common center of mass. They are always lined up and separated by distances. It shows one family of solutions, while the other two can be found by putting one of the other particles in the middle. Differential equation is a useful to know because there’s different topics within it that can help you get some answers for the most complex idea.

**References:**

* Frank, Juhan. “PHYS 7221 - The Three-Body Problem.” *Phys 7221: Classical Mechanics - Fall 2006*, 11 Oct. 2006, <https://www.phys.lsu.edu/faculty/gonzalez/Teaching/Phys7221/>.
* “Three-Body Problem.” *Wikipedia*, Wikimedia Foundation, 1 Dec. 2022, <https://en.wikipedia.org/wiki/Three-body_problem>.