① $\int_0^1 \frac{x}{\sqrt[5]{x^2+4}}\,dx$

$u = x^2+4$

$\frac{du}{dx} = 2x$

$\int_0^1 \frac{x}{\sqrt[5]{u}} \cdot \frac{1}{2x}\,du$

$= \int \frac{1}{\sqrt[5]{u}\cdot 2}\,du$

$u = x^2+4$

$x = 0$

$= 0^2+4$

$= 4$

$u = x^2+4$

$x = 1$

1^2+4

$= 5$

$\int_4^5 \frac{1}{2\sqrt[5]{u}}\,du$

$\frac{1}{2}\int_4^5 \frac{1}{u^{\frac{1}{5}}}\,du$

$\frac{1}{2}\int_4^5 \frac{u^{-\frac{1}{5}+1}}{-\frac{1}{5}+1}$

$= \frac{1}{2}\left[\frac{5u^{\frac{4}{5}}}{4}\right]_4^5$

$u=4 \quad \frac{5\cdot 4^{\frac{4}{5}}}{4} = \frac{5}{4^{\frac{1}{5}}}$

$u=5 \quad \frac{5\cdot 5^{\frac{4}{5}}}{4}$

$\Rightarrow \frac{1}{2}\left(\frac{5\cdot 5^{\frac{4}{5}}}{4} - \frac{5}{\sqrt[5]{4}}\right)$

$= \frac{5\cdot 5^{\frac{4}{5}}\sqrt[5]{4} - 20}{8\sqrt[5]{4}}$

②

② $\sum_{n=1}^{\infty} \frac{4n^6}{8n^6+8}$

$4 \cdot \sum_{n=1}^{\infty} \frac{n^6}{8n^6+8}$

$4 \cdot \sum_{n=1}^{\infty} \frac{1}{1}$

$= 4$

L'Hôpital rule
coefficient of n $\frac{n^6}{n^6} = 1$, $\Rightarrow$ diverges at 4

According to the Comparison test
this equation diverges

③ $\int 3xe^{8x}\,dx$

$3\int xe^{8x}\,dx$

$u = 8x$

$du = 8dx$

$dx = \frac{1}{8}du$

$3\int xe^{u}\frac{1}{8}du$

$\frac{8x}{8} = \frac{u}{8}$

$x = \frac{u}{8}$

$3\int \frac{u}{8}e^{u}\frac{1}{8}du$ $\Rightarrow$

$3 \cdot \int \frac{e^{u}u}{64}du$

$\int uv' = uv - \int u'v$

$u = u$

$v' = e^{u}$

$\frac{du}{du}(u) = 1$

$e^{u}+C$

$3 \cdot \frac{1}{64}(e^{u}u - \int e^{u}du)$

$\int e^{u}du = e^{u}$

$3 \cdot \frac{1}{64}(e^{8x} \cdot 8x - e^{8x})$

$\frac{3}{64}(8xe^{8x} - e^{8x}) + C$

(4) $\int \frac{3x+3}{x^2-12x+36} dx =$

factor $= x^2-12x+36 = a^2-2ab+b^2 = (a-b)^2$

$(x-6)^2$

$\int \frac{3x+3}{(x-6)^2} dx$

$(x-6)^2 \quad \frac{3x+3}{(x-6)^2} = \frac{A(x-6)^{\cancel{2}}}{\cancel{(x-6)}} + \frac{B\cancel{(x-6)}^{2}}{\cancel{(x-6)}}$

$3x+3 = A(x-6) + B$

$x=6 = \quad 3(6)+3 = A(6-6) + B$

$= 21 = 0+B$

$= 21 = B$

$\therefore B = 21$

$B=21 \Rightarrow \quad 3x+3 = A(x-6) + 21$

$x=1 \quad 3(1)+3 = A(1-6) + 21$

$3+3 = A(-5) + 21$

$6-21 = A \cdot -5$

$\frac{+15}{+5} = \frac{A(-5)}{5}$

$A = 3$

$$\int \frac{3x+3}{(x-6)^2}\,dx = \int \frac{3}{(x-6)}\,dx + \int \frac{21}{(x-6)^2}\,dx$$

$$3\int \frac{1}{(x-6)}\,dx + 21\int \frac{1}{(x-6)^2}\,dx$$

Similar to $\int \frac{1}{u}\,du = \ln|u| + c$

$$3\ln|x-6| + 21\int \frac{1}{(x-6)^2}\,dx$$

let $u = (x-6)$

$du = 1$

$$21\int \frac{1}{u^2}\,dx \Rightarrow 21\int \frac{u^{-2+1}}{-1} = 21\int \frac{(x-6)^{-1}}{-1}$$

$$21 \times \frac{(x-6)^{-1}}{-1} \Rightarrow -\frac{21 \cdot 1}{1(x-6)^{1}} = -\frac{21}{(x-6)}$$

$$3\ln|x-6| - \frac{21}{(x-6)} + C$$

(5) $\int_{-2}^{\infty} e^{-7x}\,dx$

$u = -7x$

$\frac{d}{dx}(-7x)$

$= -7\frac{d}{dx}(x)$

$\Rightarrow -7 \cdot 1$

$= -7$

$du = -7dx$

$\int e^{u}(-\frac{1}{7})du$

$\int -\frac{1}{7}e^{u}du$

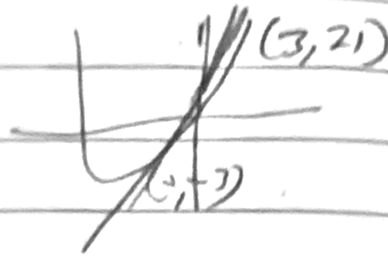
$= -\frac{1}{7}e^{-7x}\Big|_{-2}^{\infty} + C$

$-\frac{1}{7}e^{-7(\infty)} - \left|-\frac{1}{7}e^{-7(-2)}\right.$

$-0 + \frac{1}{7}e^{-7(-2)}$

$= \frac{e^{14}}{7}$ $\therefore$ It converges.

① $y = x^2 + 4x \quad y = 6x + 3$



$x^2 + 4x = 6x + 3$

$0 = -x^2 - 4x + 6x + 3$

$0 = -x^2 + 2x + 3$

$(-1, 3)\ x$

Area $= \int_{-1}^{3} \left[x^2 + 4x - 6x + 3 \right] dx$

$= \int_{-1}^{3} x^2 - 2x + 3 \ dx$

$= \int_{-1}^{3} \frac{x^3}{3} - \frac{\cancel{2}x^2}{\cancel{2}} + 3x$

$= \left[\frac{x^3}{3} - x^2 + 3x \right]_{-1}^{3}$

$\frac{3^3}{3} - 3^2 + 3(3) - \frac{-1^3}{3} - (-1)^2 + 3(-1)$

$9 - 9 + (9) - \frac{-1}{3} - 1 - 3$

$9 + \frac{1}{3} + 1 + 3$

$= \frac{40}{3}$ //

8) $\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{n^2 \cdot 2^n}$

$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L$

Ratio test

$\left| \frac{(x-1)^{n+1}}{(n+1)^2 \cdot 2^{n+1}} \div \frac{(x-1)^n}{n^3 2^n} \right|$

$\left| \frac{(x-1)^{\cancel{n+1}}}{(n+1)^2 \cdot 2^{\cancel{n+1}}} \times \frac{n^2 \cancel{2^n}}{\cancel{(x-1)^n}} \right|$

$L = \lim_{n\to\infty} = \left| \frac{n^2 (x-1)}{2(n+1)^2} \right|$

$\left| \frac{(x-1)}{9} \right| \lim_{n\to\infty} \left(\left| \frac{n^2}{(n+1)^2} \right| \right)$

$x+1 < 2$

$R = 2$ This Convergence

$(-1-2, -1+2)$

$(-1, 3)$

$-1 \leq x \leq 3$

$f(x) = e^{6x}$ $a = 3$

9) $T_3(x)$??

$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 \ldots$

$T(3) = e^{6\cdot 3} = e^{18}$

$= e^{18} + \frac{\frac{d}{dx}(e^{6x})(3)}{1!}(x-3) + \frac{\frac{d^2}{dx^2}(e^{6x})(3)}{2!}(x-3)^2 + \frac{\frac{d^3}{dx^3}(e^{6x})(3)}{3!}(x-3)^3$

$= e^{18} + \frac{6e^{18}}{1!}(x-3) + \frac{36e^{18}}{2!}(x-3)^2 + \frac{216e^{18}}{3!}(x-3)^3 \ldots$

$1! = 1$

$2! = 2 \cdot 1$ $\quad = e^{18} + 6e^{18}(x-3) + 18e^{18}(x-3)^2 + 36e^{18}(x-3)^3$ //

$3! = 1 \cdot 2 \cdot 3$

(a) $\int \frac{7}{x^2\sqrt{x^2+25}}\,dx$

$7\int \frac{1}{x^2\sqrt{x^2+25}}\,dx$

$a\sec(a)$

$\frac{dx}{du} = 5\sec^2(u)$

$\frac{du}{du} = (5\tan(u))$

$5 \cdot \frac{d}{du}(\tan(u))$

$5\sec^2(u)$

$= \int \frac{1}{(5\tan(u))^2\sqrt{(5\tan(u))^2+25}} \cdot 5\sec^2(u)\,du$

$\Rightarrow \frac{1}{5^2\tan^2(u)\sqrt{(5\tan(u))^2+25}}$

$\sqrt{(5\tan(u))^2+25} = 5\sqrt{\tan^2(u)+1}$

$\sqrt{25\tan^2(u)+25}$

$\Rightarrow \frac{1}{5^2 \cdot 5\tan^2(u)\sqrt{\tan^2(u)+1}}$

$5^2\tan^2(u) \cdot 5\sqrt{\tan^2(u)+1}$

$= \tan^2(u) \cdot 5^3\sqrt{\tan^2(u)+1} \Rightarrow$

$= \frac{1}{5^3\tan^2(u)\sqrt{\tan^2(u)+1}}$

$\Rightarrow 5 \cdot \frac{1}{5^3\tan^2(u)\sqrt{\tan^2(u)+1}}\sec^2(u)$

$= \frac{1 \cdot 5\sec^2(u)}{\tan^2(u) \cdot 5^3\sqrt{\tan^2(u)+1}}$

$= \frac{\sec^2(u)}{25\tan^2(u)\sqrt{\tan^2(u)+1}}$

$\tan^2(x)+ = \sec^2(x)$

$25\sqrt{\sec^2(u)}\tan^2(u)$

$= \frac{\sec^2(u)}{25\tan^2(u)\sqrt{\sec^2(u)}}$

$= \frac{\sec(u)}{25\tan^2(u)}$

$= 7 \cdot \int \frac{\sec(u)}{25\tan^2(u)}\,du$

$= 7 \cdot \frac{1}{25}\int \frac{\sec(u)}{\tan^2(u)}$

$\sec(x) = \frac{1}{\cos(x)}$

$\tan = \frac{\sin}{\cos}$

PTO $\Rightarrow$

$$\frac{\frac{1}{\cos u}}{\left(\frac{\sin u}{\cos u}\right)^2} = \frac{\cos(u)}{\sin^2(u)}$$

$$\Rightarrow 7 \cdot \frac{1}{25} \cdot \int \frac{\cos(u)}{\sin^2(u)}\, du$$

U substitution

$v = \sin(u)$

$\frac{dv}{du} = \cos(u)$

$dv = \frac{1}{\cos(u)}\, dx$

$$= \int \frac{\cos(u)}{v^2} \cdot \frac{1}{\cos(u)}\, dx$$

$$\frac{\cos(u)}{v^2} \cdot \frac{1}{\cos(u)}$$

$$= \frac{\cos(u) \cdot 1}{v^2 \cos(u)}$$

$$= \int \frac{1}{v^2}\, dv$$

$$= 7 \cdot \frac{1}{25} \cdot \int \frac{1}{v^2}\, dx$$

Power rule

$$7 \cdot \frac{1}{25} \cdot \frac{v^{-2+1}}{-2+1}$$

$$7 \cdot \frac{1}{25} \cdot \frac{\sin^{-2+1}(u)}{-2+1}$$

$u = \arctan\left(\frac{1}{5}x\right)$

$$7 \cdot \frac{1}{25} \cdot \frac{\sin^{-2+1}\left(\arctan\left(\frac{1}{5}x\right)\right)}{-2+1}$$

$$\sin(\arctan(x)) = \frac{x}{\sqrt{1+x^2}}$$

$$\Rightarrow 7 \cdot \frac{1}{25} \cdot \frac{\left(\frac{\frac{1}{5}x}{\sqrt{1+\left(\frac{1}{5}x\right)^2}}\right)^{-2+1}}{-2+1}$$

$$\Rightarrow -\frac{7\sqrt{25+x^2}}{25x} + C$$

11

an=

(11) $\sum_{n=1}^{\infty} (-1)^n \frac{8}{9n+7}$

Alternating test

$a_n = \frac{8}{9n+7}$

$a_n = 0$

$\lim_{n \to \infty} \left(\frac{8}{9n+7} \right)$

$8 \lim_{n \to \infty} \left(\frac{1}{9n+7} \right)$

$8 \cdot \frac{1}{\infty} = 0$

It converges due to the alternating test