$$\int \frac{-6x^3}{\sqrt{x^2+9}}\,dx$$

$x^2+9 = x^2+3^2$

$a = a\tan(\theta)$

$a = 3$

$x = 3\tan(\theta)$

$dx = 3\sec^2(\theta)\,d\theta$

$$\int \frac{-6x^3}{\sqrt{x^2+9}}\,dx = \int \frac{-6(3\tan(\theta))^3\,(3\sec^2(\theta)\,d\theta)}{\sqrt{(3\tan(\theta))^2+9}}$$

$$= \int \frac{-6\cdot 3^3 \tan^3(\theta)\cdot 3\sec^2(\theta)\,d\theta}{\sqrt{9\cdot(\tan^2(\theta)+1)}}$$

$$= \int \frac{-6\cdot 3^4 \tan^3(\theta)\sec^2(\theta)\,d\theta}{\cancel{3}\sqrt{\sec^2(\theta)}}$$

$$= \int \frac{-6\cdot 3^3 \tan^3(\theta)\sec^2(\theta)\,d\theta}{\sec(\theta)}$$

$$= \int -6\cdot 3^3 \tan^3(\theta)\sec(\theta)\,d\theta$$

Trignumetric Integral

$\frac{d}{d\theta}\tan(\theta) = \sec^2(\theta)$

$= \sec(\theta)\tan(\theta)$

$\tan^2(\theta)+1 = \sec^2(\theta)$

$\tan\theta = \sec^2(\theta)-1$

$$\int -6\cdot 3^3 \tan^2(\theta)\cdot\sec(\theta)\tan(\theta)\,d\theta$$

$$= \int -6\cdot 3^3(\sec^2(\theta)-1)\sec\theta\tan\theta\,d\theta$$

let $u = \sec\theta$

$du = \sec\theta\tan\theta\,d\theta$

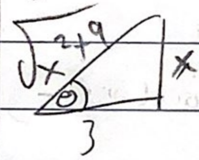
$$= \int -6\cdot 3^3(u^2-1)\,du$$

$$= -162\left(\frac{u^3}{3} - u\right) + c$$

$$= 162\left(\frac{\sec^3(\theta)}{3} - \sec(\theta)\right) + c$$

$x = 3\tan(\theta)$

$\frac{x}{3} = \tan(\theta) = \frac{O}{A}$



$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{H}{A} = \frac{\sqrt{x^2+9}}{3}$

$$= 162\left(\frac{\left(\frac{\sqrt{x^2+9}}{3}\right)^3}{3} - \frac{\sqrt{x^2+9}}{3}\right) + C$$

$$= -162\left(\frac{(\sqrt{x^2+9})^3}{81} - \frac{\sqrt{x^2+9}}{3}\right) + C$$