

**Sections 3.8, 5.2, 6.1 and 6.2 problems 1-9****Exam is 1 hour 15 minutes.****Ok:** handwritten notes, calculator (TI 83/84)**Not ok:** printouts, book, **TI 89**, laptop, tablet, cell phone or other handheld

1. (20 pts) Given an RLC circuit with  $R=2$  ohms,  $L=1/10$  henries,  $C=1/260$  farads and  $E(t)=\sin 60t$ ; find the
  - a. differential equation for the current  $I$  (remember to use  $E'(t)$ , not  $E(t)$ );
  - b. the form of the transient current  $I_p$ ;
  - c. the periodic current  $I_h$ ;

2. (20 pts) Given a mass spring damper with  $m=1$  kg,  $\gamma=2$ Ns/m,  $k=50$ N/m,  $F=20\cos(\omega t)$ ,
  - a. find the value of  $\omega$  that gives practical resonance;
  - b. if the damper is removed, then the circular frequency without a forcing function is  $5\sqrt{2}$ . Suppose that the forcing function has circular frequency  $7\sqrt{2}$  and that the initial conditions give a solution

$$y(t) = \cos 5\sqrt{2}t - \cos 7\sqrt{2}t$$

Use the appropriate sum or difference identities to write the solution as a product of 2 trigonometric functions of different frequencies. Sketch a graph over one long period to illustrate the phenomenon of beats.

3. (20 pts) For  $y'' - xy' - y = 0$ ,  $x_0 = 0$ 
  - a. find the recurrence relation of a power series solution.
  - b. Use the relation to find the first 3 terms of one of the 2 basic solutions.

$$4. (20 \text{ pts}) \text{ Given } f(t) = \begin{cases} t, & [0, 1) \\ 2-t, & [1, 2) \\ 0, & [2, \infty) \end{cases}$$

- a. Graph  $f(t)$  over  $[0, 3]$
  - b. classify as continuous, piecewise continuous, or neither
  - c. use the definition to find the Laplace Transform
5. (20 pts) Use the table of Laplace Transforms given in class to find the inverse Laplace Transform of

$$Y(s) = \frac{5s^2 + 3s + 30}{s^3 + 2s^2 + 10s}$$