Solve the IVP $y' = \dfrac{1-2x}{y}; \quad y(1) = -2$. Find the interval of validity (domain).

Separating variables and integrating, we get:

$\int y\,dy = \int 1 - 2x\,dx$ or $y^2 / 2 = x - x^2 + k$

$y^2 = 2x - 2x^2 + c$ or $y = \pm\sqrt{2x - 2x^2 + c}$

To find c, we substitute in the IV:

$(-2)^2 = 2(1) - 2(1)^2 + c$ or $c = 4$

We need to decide which branch to take. To have a negative initial value, we need the lower branch:

$y = -\sqrt{2x - 2x^2 + 4}$

To find the interval of validity, we need the radicand to be nonnegative:

$2x - 2x^2 + 4 \geq 0$

Multiplying by −1, dividing by 2 and factoring:

$x^2 - x - 2 = (x+1)(x-2) \leq 0$

This is parabola (up) with zeros at −1 and 2 so [−1,2] is the domain. However, at the endpoints $y = 0$ and we will have vertical tangent lines, so we exclude them. Thus the interval of validity is (−1,2). Not part of the quiz, but as confirmation of our analysis, we look at the slope field. Note in particular all the settings: