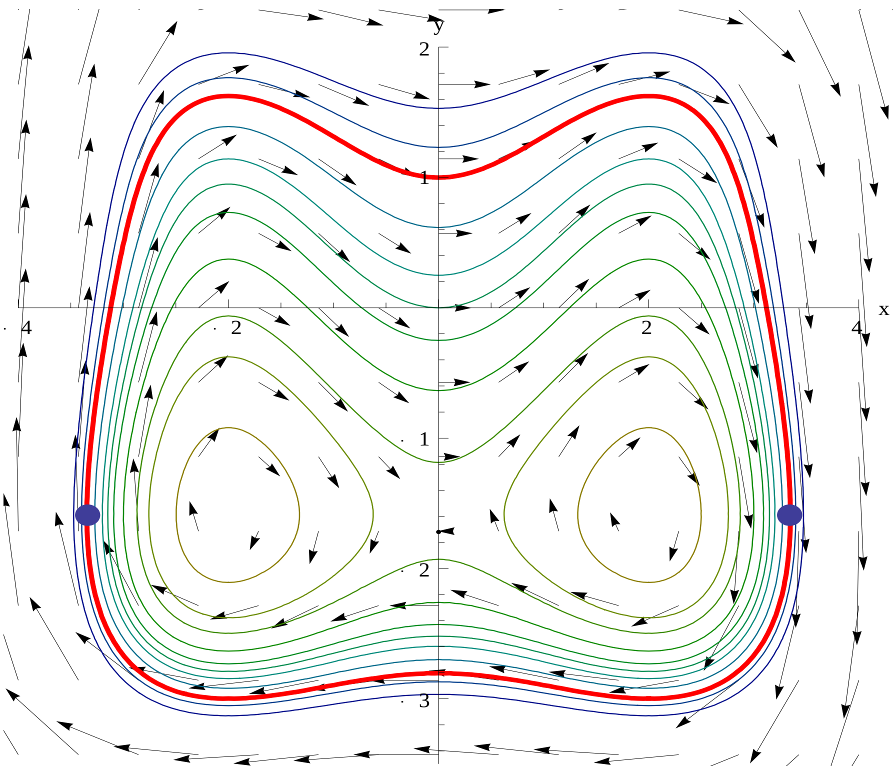
Exam is 1 hour 30 minutes. Starts at 2:15, ends at 3:45
Ok: handwritten notes, calculator (TI 83/84) **Not ok**: printouts, book, **TI 89,** laptop, tablet, cell phone or other handheld

Part I Chapters 1 and 2 (30%, 25 min)

A. There are 2 theorems which apply to first order ODE's. For the linear equation $y' + p(t)y = q(t)$ as long as the function(s)________ is (are) continuous, a solution always exists and is unique. In fact, we can often determine a minimal interval of validity. For instance, for $y' - \frac{3}{t^2-9}y = \frac{1}{t^2}; \quad y(-1) = 2$, the solution has ____________ as its interval of validity. For a nonlinear IVP $y' = f(t, y), \quad y(t_0) = y_0$, a solution always exists and is unique provided that ________ and the partial derivative of ________ are _______________ at the point _____________. For example, a solution will always exist and be unique for the nonlinear equation $y' = \frac{4x - x^3}{4 + y^3}$ provided that $y \neq$_____. The interval of validity is much more complicated as it will vary for each initial value. The slope field for the same equation is given to the left. For the IV (0,−2.8), **darken the solution on the graph**. The approximate interval of validity for the solution is _____________. For the IV (2,−1), **darken its solution on the graph**. The approximate interval of validity of this 2nd solution is ____________.



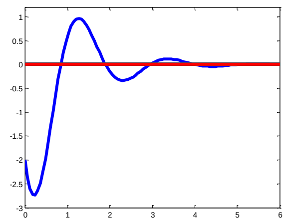
B. For fluid stability equation: $y' = (\cos t + 2)y - y^3$, identify type and provide 1st few steps of sol'n as follows:

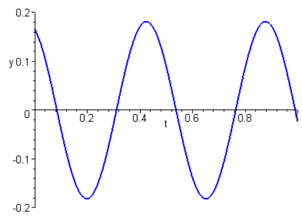
- **Separable:** separate variables and express as integrals. Note what integration technique to use to evaluate the integrals (integration by parts, partial fractions, etc.).
- **Homogeneous:** perform the substitution and separate the variables.
- **Linear:** put in standard form and find and apply the IF μ. Express y as an integral. Note what integration technique to use to evaluate the integrals (integration by parts, partial fractions, etc.).
- **Bernoulli:** perform the substitution and put resulting linear equation into standard form.
- **Exact:** show exactness, integrate M with respect to x to find Ψ up to "constant" C(y)

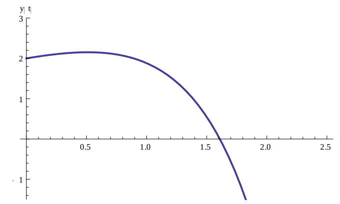
Part II Chapters 3 and 6 (40%, 35 min)

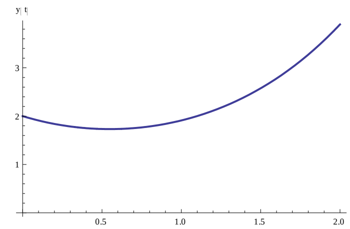
Match the following by drawing a line from each item in a row to an item in the row below.
Categories are i. graph, ii. sol'n, iii. short term analysis iv. long term analysis.

i.









ii. $y(t) = 1/2\, e^{t} + 3/2\, e^{-t}$ $\quad y(t) = e^{-t}(-2\cos(3t) - 3\sin(3t))$ $\quad y = \frac{1}{6}\cos 8\sqrt{3}\,t - \frac{1}{8\sqrt{3}}\sin 8\sqrt{3}\,t$ $\quad y(t) = -1/2\, e^{3t/2} + 5/2\, e^{t/2}$

iii. Solution is oscillatory but damped. At first the solution has exponential decay. Solution is purely oscillatory. At first the solution increases.

iv. Function goes to 0 as t goes to ∞. Function has no limit as t goes to ∞. The exponential growth term eventually takes over and the function goes to ∞ as t goes to ∞. Term with negative coefficient takes over and function goes to -∞ as t goes to ∞.

Given the RLC circuit with resistance 4 Ohms, inductance 1 Henry and capacitance 0.025 Farads

A. but with no external electromotive force,
   1. find the form of the solution
   2. find natural and quasi periods and use example to discuss the effect of damping on period

B. for the external forcing function E(t) = cos(6t), using the method of undetermined coefficients, find the form of a particular solution

C. for y(0) = 40 and y'(0) = 0 and the External forcing function $40u_3(t)$, use the Laplace transform methods to solve.

Part III Chapter 5 (15%, 15 min) find the recursive formula and the first few terms of a series solution to

$$(t+1)y'' - ty' - y = 0, \quad t_0 = 0$$

Part IV Chapter 8 (15%, 15 min) Write down the pseudo-code for the improved Euler method with h=.05 and final value t = 2. Perform the first 2 iterations by hand and compare those values with the exact solution.

$$y' = .5 - t + 2y, \quad y(0) = 1$$