Here are the solutions to p. 174 #23-26 of the text:

23. Set $y_2(t) = t^2\, v(t)$. Substitution into the differential equation results in

$$t^2(t^2v'' + 4tv' + 2v) - 4t(t^2v' + 2tv) + 6t^2v = 0\,.$$

After collecting terms, we end up with $t^4v'' = 0$. Hence $v(t) = c_1 + c_2t$, and thus $y_2(t) = c_1t^2 + c_2t^3$. Setting $c_1 = 0$ and $c_2 = 1$, we obtain $y_2(t) = t^3$.

24. Set $y_2(t) = t\,v(t)$. Substitution into the differential equation results in

$$t^2(tv'' + 2v') + 2t(tv' + v) - 2tv = 0\,.$$

After collecting terms, we end up with $t^3v'' + 4t^2v' = 0$. This equation is linear in the variable $w = v'$. It follows that $v'(t) = ct^{-4}$, and $v(t) = c_1t^{-3} + c_2$. Thus

$y_2(t) = c_1t^{-2} + c_2t$. Setting $c_1 = 1$ and $c_2 = 0$, we obtain $y_2(t) = t^{-2}$.

25. Following the reduction of order technique given, $y_1 = 1/t$, $p(t) = 3/t$, so the equation for v is $v''/t + v'/t^2 = 0$. After separating the variables the equation becomes $v''/v' = -1/t$, so $\ln v' = -\ln t + c$. We obtain that $v' = c/t$ and then $v = c\ln t$. Thus the second solution is $y_2 = \ln t/t$.

26. Set $y_2(t) = tv(t)$. Substitution into the differential equation results in $v'' - v' = 0$. This equation is linear in the variable $w = v'$. It follows that $v'(t) = c_1e^t$, and $v(t) = c_1e^t + c_2$. Thus $y_2(t) = c_1te^t + c_2t$. Setting $c_1 = 1$ and $c_2 = 0$, we obtain $y_2(t) = te^t$.