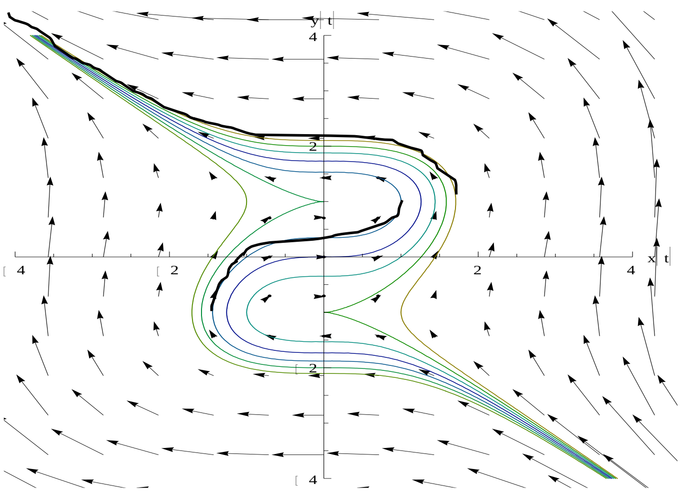
Exam is 1 hour 15 minutes.

Ok: handwritten notes, calculator (TI 83/84) **Not ok**: printouts, book, **TI 89,** laptop, tablet, cell phone or other handheld

Part I (30%, 10 min) ~ quizzes: fill in blank, short answer, T-F, matching. Emph: defs, thms & analysis using slope field.

There are 2 theorems which apply to first order ODE's. For a linear equation, as long as the coefficient functions are continuous, a solution always__**exists**_ and is ___**unique**___. In fact, the validity of solutions will extend until a discontinuity in the coefficient functions is reached. For instance, for $y'-3/(t^2-4)y=1/t^2;\quad y(1)=3$, the solution has ___**(0, 2)**________ as its interval of validity. For a nonlinear equation $y'=f(t,y)$, a solution always___**exists**__ and is _____**unique** ____provided that f and the _**partial**__ __**derivative**_ (2 words) of f are _**continuous**__ around the IV. A slightly weaker theorem holds if we have __**continuity**_ of f but not of its ___**partial**__ __**derivative** ___ (2 words). In that case, we get __**existence**_ but not _**uniqueness**__. For the nonlinear equation $\dfrac{dy}{dx}=\dfrac{x^2}{1-y^2}$ a solution will always ____**exists**_ and is ___**unique** _____ provided that ____ $y\neq\pm 1$_____. The interval of validity is much more complicated as it will vary for each initial value. The slope field for the same equation is given to the left. For the IV (0,0), darken the solution on the graph. The approximate interval of validity for the solution is ____**(−1, 1)**____. For the IV (0,2), darken its solution on the graph. The approximate interval of validity for its solution is _**(−∞, 1.75)**__.



We discuss the limiting behavior, assuming that the solutions follow the direction of the arrows.

For any solution whose initial value is in Q1 and has y>1: It is repelled from $y=1$, $y=-x$. (circle one) and it is attracted to $y=1$, $y=-x$ (circle one). $\lim\limits_{x\to-\infty} y=$ _$-\infty$__.

Part II (15%, 15 min) Linear first order IVP with nonconstant coefficients (explicit sol'n)

$$ty'+2y=t^2-t+1;\quad y(1)=1/2$$
$$y'+\left(2/t\right)y=t-1+1/t$$
$$\mu=e^{\int 2/t\,dt}=t^2$$
$$t^2y'+2ty=t^3-t^2+t$$
$$t^2y=t^4/4-t^3/3+t^2/2+c$$
$$y=t^2/4-t/3+1/2+ct^{-2}$$
$$y(1)=1/4-1/3+1/2+c=1/2$$
$$c=1/12$$
$$y=t^2/4-t/3+1/2+1/(12t^2)$$

Part III (15%, 15 min) Exact equation which requires an I.F, like p 134 #31. (1st verify exactness, implicit sol'n)

$$3x^2y + y^2 + \left(2x^3 + 3xy\right)y' = 0$$

$$M_y = 3x^2 + 2y \neq 6x^2 + 3y = N_x \text{ so not exact}$$

There are 2 formulas to try to find the I.F.

$$\frac{d\mu}{dx} = \frac{M_y - N_x}{N}\mu. \qquad \frac{d\mu}{dy} = \frac{M_y - N_x}{M}\mu$$

Is the coefficient of μ in the 1st equation a function of x?

$$\frac{M_y - N_x}{N} = \frac{3x^2 + 2y - \left(6x^2 + 3y\right)}{2x^3 + 3xy} = \frac{-3x^2 - y}{2x^3 + 3xy} = \frac{-(3x^2 + y)}{x(2x^2 + 3y)}$$

No, it is not! Next, we see if the coefficient of μ in the 2nd equation is a function of y:

$$\frac{M_y - N_x}{M} = \frac{-(3x^2 + y)}{3x^2y + y^2} = \frac{-(3x^2 + y)}{y(3x^2 + y)} = \frac{-1}{y}$$

Yes! Actually, we only need the IF up to a constant, so we solve

$$\frac{d\mu}{dy} = \frac{1}{y}\mu \Rightarrow \mu = y$$

Multiplying the original equation by the IF we get:

$$3x^2y^2 + y^3 + \left(2x^3y + 3xy^2\right)y' = 0$$

$$M_y = 6x^2y + 3y^2 = N_x \text{ so exact!}$$

Integrating M with respect to x, we get

$$\psi = x^3y^2 + xy^3 + C(y)$$

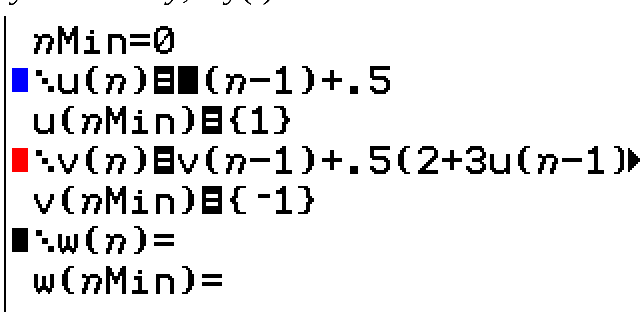
$$\psi_y = 2x^3y + 3xy^2 + C'(y) = 2x^3y + 3xy^2$$

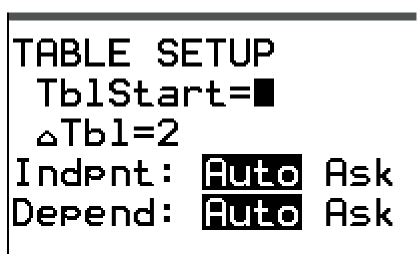
So we can set $C(y) = 0$ and the implicit solution is:

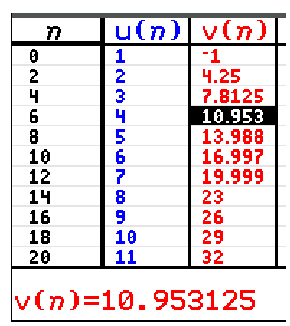
$$x^3y^2 + xy^3 = c$$

Part IV (10%, 10 min) Euler method for numerical solution: $h = 0.5$; t final is 4. Be sure to provide a screen shot for the "y=" window in "seq" mode as well a table showing the values of *t* and *y* for integer values of *t*.

$$y' = 2 + 3t - y; \quad y(1) = -1$$







Part V (30%, 20 min) 3 1st order eqtns. Identify type, provide 1st few steps for sol'n, & check soln (will be given).
p. 133, problems 4, 30 and 32. See openlab "part V exam 1" for solutions (actually only **some** of the solutions).

4. Equation is linear:

$$y' = 3 - 6x + y - 2xy$$

$$y' + (2x - 1)y = 3 - 6x$$

It is now in standard form. Following the directions from the document "part V exam 1". We need to find the IF.

$$\mu = e^{\int 2x - 1 dx} = e^{x^2 - x}$$

Multiplying by IF and integrating, we get

$$e^{x^2-x}y=\int(3-6x)e^{x^2-x}dx=-3\int(2x-1)e^{x^2-x}dx=-3e^{x^2-x}+c \text{ or } y=-3+ce^{x-x^2}$$

Actually, you were only asked to express answer as an integral:

$$y=e^{x-x^2}\int(3-6x)e^{x^2-x}dx$$

30. Note how the degree of every term is 2, which is why the equation is homogeneous. Solving for y':

$$y'=\frac{3y^2+2xy}{2xy+x^2}=\frac{3(y/x)^2+2(y/x)}{2(y/x)+1}.$$

Substituting $u=y/x$ gives that $y=ux$ and then

$$y'=u'x+u=\frac{3u^2+2u}{2u+1},$$

which implies that

$$u'x=\frac{3u^2+2u}{2u+1}-u=\frac{u^2+u}{2u+1},$$

Completing the variable separation, your final answer is:

$$\frac{2u+1}{u^2+u}=\frac{dx}{x}$$

32. Equation is Bernoulli with $n = 2$:

$$xy'+y-y^2e^{2x}=0 \text{ or } y'+(1/x)y=\left(e^{2x}/x\right)y^2$$

According to the instructions, we need to make the substitution and put the new ODE into standard linear form. When $n = 2$, our substitution formula boils down to $y=v^{-1}$; $y'=-v^{-2}v'$. Making the substitution and then multiplying by $-v^2$, we get

$$-v^{-2}v'+(1/x)v^{-1}=\left(e^{2x}/x\right)v^{-2} \text{ or } v'-(1/x)v=-e^{2x}/x,$$

which is linear in standard form. Hence, the prescribed tasks have been completed.