- You may use a scientific or graphing calculator. No use of computer software.
- At the end of class, be sure to turn in your formula sheet (1 sheet, 2 pages, hand-written), worth 10%.

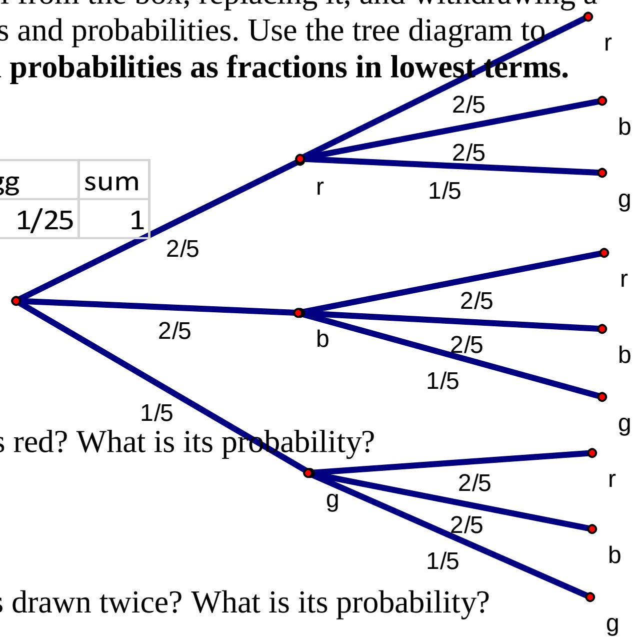
1. The formula **=B$2*A2** is located in cell **B1**. (10%)
   a) What does cell B1 evaluate to? **3*2=6**
   b) If this was copied and pasted into cell D3, what would resulting formula be? **=D$2*C4**

	A	B	C	D
1	2	**=B$2*A2**	4	5
2	3	3	8	6
3	5	4	3	?????????
4	4	3	4	9

There will be problems similar to 4 of the following 10 problems, each will be worth 20 points.

2. Consider an experiment that consists of withdrawing a ball from the box, replacing it, and withdrawing a second ball. Draw a tree diagram. Be sure to include labels and probabilities. Use the tree diagram to make a table with outcomes and probabilities. **Express all probabilities as fractions in lowest terms.** There are 2 red, 2 blue and 1 green ball in the box.

x	rr	rb	rg	br	bb	bg	gr	gb	gg	sum
P(x)	4/25	4/25	2/25	4/25	4/25	2/25	2/25	2/25	1/25	1



a. What is the sample space of this experiment?
1st row of table above

b. As a set, what is the event A: the first ball drawn is red? What is its probability?

rr	rb	rg
4/25	4/25	2/25

A={rr,rb,rg}, P(A)=10/25=2/5

c. As a set, what is the event B: the same color ball is drawn twice? What is its probability?

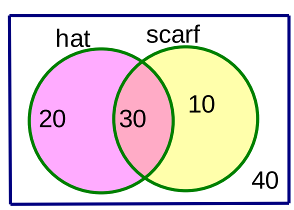
rr	bb	gg
4/25	4/25	1/25

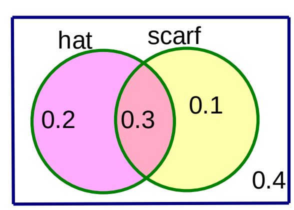
B={rr, bb, gg}, P(B) = 9/25

d. Are events A and B independent? **A ∩ B={rr} so**
P(A ∩ B)=4/25=0.16≠P(A)*P(B)=2/5*9/25=18/125=0.144 Hence, dependence.
If A or B occurs, it has a negative impact on the chance of the other event.

3. To get credit, you must use a Venn Diagram: At a Black Lives Matter march in the late fall were 100 participants, 40 students brought neither a scarf nor a hat, 50 brought a hat, and 40 brought a scarf. If one of them was randomly chosen, find the probability that he or she brought

(a) A scarf or a hat
We are given that $P(A^C \cap B^C)= P((A\cup B)^C)=0.4$.
Therefore $P(A\cup B)=1-0.4=0.6$



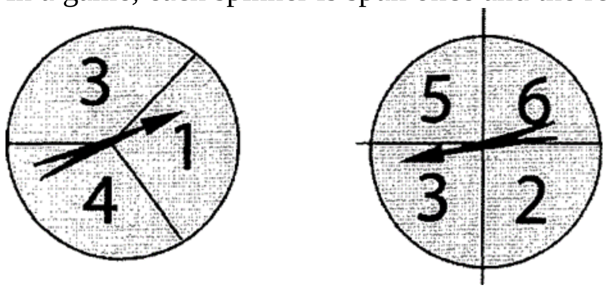


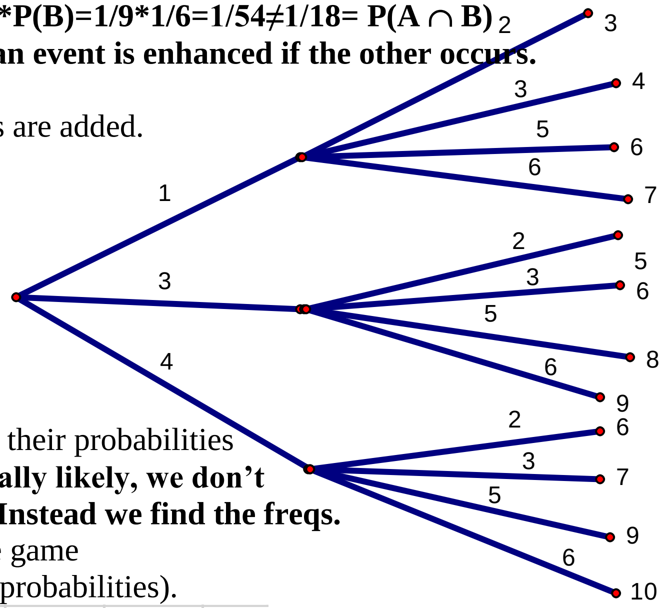
(b) A scarf and a hat
$P(A\cap B)=P(A)+P(B) - P(A\cup B)=0.5+0.4-0.6=0.3$

(c) Are the events student brings a hat and student brings a scarf independent?
0.3=P(A∩B)=P(A)*P(B)= 0.5*0.4=0.2 Hence, dependence: this time, mutual effect is positive, i.e., a person with a scarf is more likely to be a person with a hat than not and vice versa.

4. 2 fair 6-sided die are rolled (one green & one red) & the outcome is coordinate (green face, red face).
   a. Let A be the event that the faces sum to an even number greater than 8. Find P(A).
      A={46,55,64,66} so P(A)=4/36=1/9
   b. Let B be the event that the faces are the same (doubles). Find P(B).
      B=**{11,22,33,…,66} so P(B)=6/36=1/6**
   c. Find $A \cap B$ as set. Find $P(A \cap B)$.
      $A \cap B$={55,66} so $P(A \cap B)$=2/36=1/18
   d. Are A and B independent events? **No, P(A)*P(B)=1/9*1/6=1/54≠1/18= $P(A \cap B)$**
      Events A & B are dependent. Chance of an event is enhanced if the other occurs.

5. In a game, each spinner is spun once and the results are added.





   a. Use a tree to find paths to the outcomes and their probabilities
      Since the outcomes for each path are equally likely, we don't need to use the tree to find probabilities. Instead we find the freqs.
   b. Find the random variable that represents the game (make a table with the possible outcomes and their probabilities).

x	3	4	5	6	7	8	9	10	SUM
f(x)	1	1	1	3	2	1	2	1	12
P(x)	1/12	1/12	1/12	1/4	1/6	1/12	1/6	1/12	1

6. Three cards are pulled from a deck of 52 cards. Find the probability of obtaining
   a. at least one club. Find complement (no club)

$$1-\frac{\binom{39}{3}}{\binom{52}{3}}$$

   b. a pair. choose face value of pair, then suits of pair cards, then the nonpair card,

$$\frac{13\binom{4}{2}\binom{48}{1}}{\binom{52}{3}}$$

   c. 3 of a kind: almost the same reasoning as for pair

$$\frac{13\binom{4}{3}}{\binom{52}{3}}$$

   d. straight (3 cards whose face values are in order, an ace can be lower than 2 or higher than king. Choose starting face (all except Q) & suits for each card

$$\frac{12*4^3}{\binom{52}{3}}$$

   e. a flush (all 3 of the same suit) choose suit and then face values for the 3 cards

$$\frac{4\binom{13}{3}}{\binom{52}{3}}$$

   f. a straight flush: choose suit & starting face

$$\frac{4*12}{\binom{52}{3}}$$

f. use your answers to d, e, f to determine whether getting a straight and getting a flush are independent events. Does d*e=f?

$$\frac{12*4^3}{\binom{52}{3}}\frac{4\binom{13}{3}}{\binom{52}{3}}=?\frac{4*12}{\binom{52}{3}}$$ **Extract expressions from the RHS that are on LHS and see if what's left evaluates to 1.**

$$\frac{\binom{13}{3}4^3}{\binom{52}{3}}\frac{4*12}{\binom{52}{3}}=?\frac{4*12}{\binom{52}{3}}$$ or is $$\frac{\binom{13}{3}4^3}{\binom{52}{3}}=?1$$ **Evaluating** $$\frac{13*12*11*4^3}{52*51*50}=\frac{6*11*4^2}{51*25}$$ = **0.83 whose reciprocal is ~6/5.**

Hence, if a flush or straight occurs the chance of the other increases by about 20%. By the way, d*e~0.18% and f~0.22%.

7. A jar contains 3 chocolate chip cookies and x oatmeal cookies. Two cookies are pulled one at a time from the jar without replacement.
   a. Find an expression that represents the probability one cookie is chocolate chip and the next cookie is oatmeal.

$$\frac{3}{3+x}*\frac{x}{2+x}=\frac{3x}{(3+x)(2+x)}$$

   b. Find an expression that represents the probability one cookie is chocolate chip and the other cookie is oatmeal, regardless of the order in which they come out.

$$\frac{3}{3+x}*\frac{x}{2+x}+\frac{x}{3+x}*\frac{3}{2+x}=\frac{6x}{(3+x)(2+x)} \text{ or } \frac{\binom{3}{1}\binom{x}{1}}{\binom{x+3}{2}}$$

   c. If the chance of getting the event described in a. is 2/7, find an equation and solve to determine x.

$$\frac{3x}{(3+x)(2+x)}=\frac{2}{7} \text{ or } 21x=2(3+x)(2+x)=12+10x+2x^2 \text{ or } 0=2x^2-11x+12=(2x-3)*(x-4) \text{ or}$$

{3/2, 4} but answer must be an integer so x=4.

8. A 5 digit PIN number can begin with any digit (except zero) and the remaining digits have no restriction, but otherwise is selected randomly.
   a. Find the probability that the PIN code has no repeated digits, begins with a 7 and ends with an 8.

The outer digits are determined but the middle 3 digits have no restriction so 10^3/(9*10^4)=1/90

   b. Find the probability the PIN code is odd: **P(O)=9*10^3*5/(9*10^4)=1/2**
   c. Find the conditional probability that the PIN code is odd given that the code has no repeated digits: **Make selections for the last and first digits which will have 5 and 8 possible choices respectively, then the number of choices for the middle 3 digits are 8,7 and 6, respectively. Denominator is enumerated by counting choices for 1st digit then it is a permutation for remaining digits: P(O|NR)=P(O∩NR)/P(NR)=5*8^2*7*6/(9^2*8*7*6)=5*8/9^2**
   d. Are the events PIN is odd and PIN has no repeated digits independent?
   No, P(O)=.5≠ P(O|NR)=40/81. Not allowing repeats has a slightly negative effect on the chance of being odd.

9. There are 12 top female runners in a marathon, 7 from Africa and 5 from outside of Africa. If they each have an equal chance getting any of the top 12 positions, find the chance that
   a. exactly 3 of the top 5 runners will be from Africa

$$P(X=3)=\frac{\binom{7}{3}\binom{5}{2}}{\binom{12}{5}}=\frac{7nCr3*5nCr2}{12nCr5}\approx 0.44$$

   b. all 5 of the top runners will be from Africa

$$P(X=5)=\frac{\binom{7}{5}\binom{5}{0}}{\binom{12}{5}}=\frac{7*6*5*4*3}{12*11*10*9*8}=\frac{7}{11*3*8}=\frac{7}{264}\approx 0.027$$

   c. at least 3 of the top 5 runners will be from Africa.

$$P(X\geq 3)=\frac{\binom{7}{3}\binom{5}{2}}{\binom{12}{5}}+\frac{\binom{7}{4}\binom{5}{1}}{\binom{12}{5}}+\frac{\binom{7}{5}\binom{5}{0}}{\binom{12}{5}}\approx 0.69$$

10. A fair coin is flipped 6 times. Let X represent the number of heads in the first 3 tosses. Let Y represent the number of heads in the 2^{nd} set of 3 tosses. Make a table of X cross Y with the marginals determined by the probabilities for X and Y. Use independence to determine the probabilities of the interior.
   a. Use the table to find P(X=Y).

The distributions for both X and Y are:

k	0	1	2	3
P(k)	1/8	3/8	3/8	1/8

Here is a table for X cross Y with an interior not yet filled in:

Y\X	0	1	2	3	
0					1/8
1					3/8
2					3/8
3					1/8
	1/8	3/8	3/8	1/8	1

We fill in the interior by using independence, e.g.,
P(X=1 ∩ Y=3)= P(X=1) *P(Y=3)=3/8*1/8=3/64
We end up with the following:

Y\X	0	1	2	3	
0	1/64	3/64	3/64	1/64	1/8
1	3/64	9/64	9/64	3/64	3/8
2	3/64	9/64	9/64	3/64	3/8
3	1/64	3/64	3/64	1/64	1/8
	1/8	3/8	3/8	1/8	1

P(X=Y)=P(0,0)+P(1,1)…+P(3,3)
=1/64+9/64+9/64+1/64=20/64=5/16

   b. Use symmetry, complementation and your answer from a. to find P(X>Y)

$$P(X>Y)=\frac{1-P(X=Y)}{2}=\frac{1-5/16}{2}=\frac{16-5}{32}=\frac{11}{32}$$

11. A grocery store obtains 35% of its produce from vendor A, and 65% of its produce from vendor B. It is expected that spoilage will result in 12% of vendor A's produce and 17% of vendor B's produce to be discarded. Find the probability a randomly picked produce item came from vendor A, given that it was picked from the discard pile.

    This is a Bayes Theorem Problem:

    Chance that a random piece of fruit is spoiled is P(S)=.35*.12+.65*.17

    P(A|S)= 35*.12/(35*.12+.65*.17)=.2754 about 2/7