NYCCT MAT2572 Halleck Fall 2017 Practice exam 2

1. Based on recent experience, 10-year-old passenger cars going through a motor vehicle inspection station have an 80% chance of passing the emissions test. Suppose that a municipality owns 200 such cars.
   a. Find the RV for the number of cars that do **not** pass by using the binomial distribution.

   b. Using your work from part a and the definition of expectation, find an expression for the expectation of the number of repairs needed.

   c. Find the expectation for the i^{th} car **not** to pass $E(X_i)$ by finding the RV that represents the i^{th} car X_i (0 for pass, 1 for not pass) and using the definition of expectation.

   d. Find the expectation for # cars that are **not** expected to pass by using your answer to c. and $E(\sum X_i) = \sum E(X_i)$.

   e. If the average cost for a repair is $300, about how much can the municipality expect to spend this year on emission repairs for these 200 cars?

2. Let $f_Y(y) = ay(4-y), \quad 0 \le y \le 3$.
   a. Find a so that $f_Y(y)$ is a PDF.

   b. Graph $f_Y(y)$ from $0 \le y \le 3$ (plot every 0.5).

   c. Find and graph $F_Y(y)$, the corresponding CDF, from $0 \le y \le 3$ (plot every 0.5).

   d. Find $P(|Y-1|) < 0.5$. Graph the inequality on your plot from part b) and shade the area representing the probability.

   e. Find the expectation and label on your plot from part b)

   f. Find the median and label on both graphs (the first with a vertical line and the second with a horizontal line and a vertical line).

   g. Compare the relative positions of the median and the expectation. Explain how the comparison relates to any skewing.

3. Consider an experiment that consists of withdrawing a ball from the box, NOT replacing it, and withdrawing a second ball. There are 2 red and 4 green balls in the box. An outcome will be a coordinate of 2 colors, say (r,r), which can be shortened to rr.
   a. What is the sample space of this experiment? Is this a random variable? Why or why not?
   b. To get a random variable W, we count the number of red balls selected. Find the possible outcomes and express their probabilities as quotients of binomial coefficients. Write W as a table with row 1 as the outcomes and row 2 the corresponding probabilities.
   c. Evaluate the binomial coefficients, leaving answers as fractions. Rewrite table representing W.
   d. Use the 2 row table of c to find the expectation $\mu=E(W)$.
   e. Find the 2 row table for W^2 and use it find $\mu_2=E(W^2)$.
   f. Use the results from d and e to find V(W).
   g. If X is an RV indicating whether the first ball is red or not (1, 0 respectively) and Y is whether the second ball is red or not (1, 0 respectively). Find E(X) and E(Y) and show that their sum corresponds to your answer in d.
   h. Find V(X) and V(Y) for X and Y described in f. and show that their sum DOES NOT correspond to your answer in d. Why not? Try to reason why it is more or less.