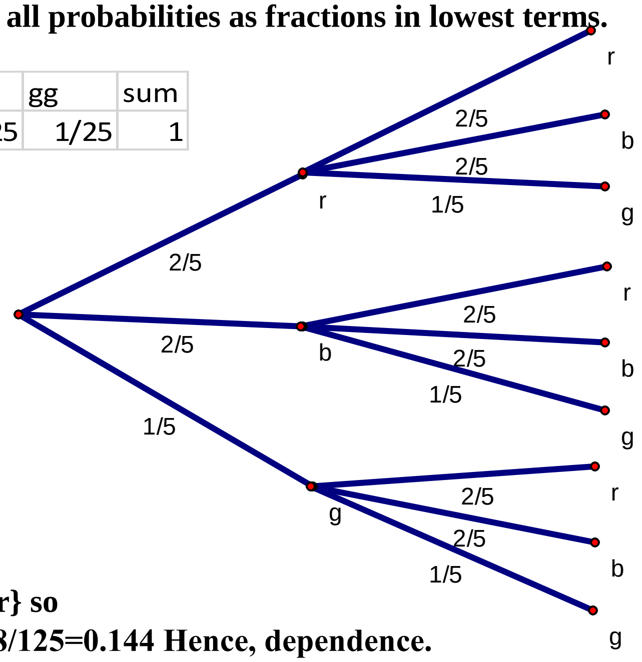
1. Consider an experiment that consists of withdrawing a ball from the box, replacing it, and withdrawing a second ball. Draw a tree diagram. Be sure to include labels and probabilities. Use the tree diagram to make a table with outcomes and probabilities. **Express all probabilities as fractions in lowest terms.** There are 2 red, 2 blue and 1 green ball in the box.



x	rr	rb	rg	br	bb	bg	gr	gb	gg	sum
P(x)	4/25	4/25	2/25	4/25	4/25	2/25	2/25	2/25	1/25	1

a. What is the sample space of this experiment?
Sample space is the first row of above table

b. As a set, what is the event A: the first ball drawn is red? What is its probability?

rr	rb	rg
4/25	4/25	2/25

A={rr,rb,rg}, P(A)=10/25=2/5

c. As a set, what is the event B: the same color ball is drawn twice? What is its probability?

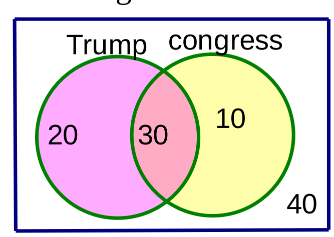
rr	bb	gg
4/25	4/25	1/25

B={rr, bb, gg}, P(B) = 9/25

d. Are events A and B independent? If not, are they positively or negatively reinforcing? **$A \cap B$={rr} so**
$P(A \cap B)=4/25=0.16 \neq P(A)*P(B)=2/5*9/25=18/125=0.144$ Hence, dependence.
P(B|A)= (4/25)/(2/5)=2/5
Hence, if A occurs, it has a positive impact on the chance of B (36% -> 40%).
Same will be true for reverse situation, so the events positively reinforce each other.

2. 100 people were present at a protest against cuts in education spending by the federal government. 40 brought placards denouncing Trump, 50 brought placards denouncing congress, and 20 did not have placards. If one of them was randomly chosen, find the probability that he or she brought

Let A = denounce Trump, let B = denounce congress



a. Trump or congress
$P(A^C \cap B^C)= P((A \cup B)^C)=0.4$. Therefore $P(A \cup B)=1-0.4=0.6$

b. Trump and congress **$P(A \cap B)=P(A)+P(B) - P(A \cup B)=0.5+0.4-0.6=0.3$**

c. Are the events student denounces Trump and denounces congress independent? If not, are they positively or negatively reinforcing?
$0.3=P(A \cap B) \neq P(A)*P(B)= 0.5*0.4=0.2$ Hence, dependence. P(A|B)=75% and P(A)=50%
Same will be true for reverse situation, so the events positively reinforce each other.

3. 2 fair 6-sided die are rolled (one green & one red) & the outcome is coordinate (green face, red face).

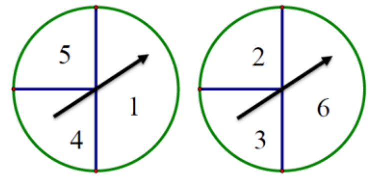
a. Let A be the event that the faces sum to an even number greater than 8. Find P(A).
A={46,55,64,66} so P(A)=4/36=1/9

b. Let B be the event that the faces are the same (doubles). Find P(B).
B=**{11,22,33,…,66} so P(B)=6/36=1/6**

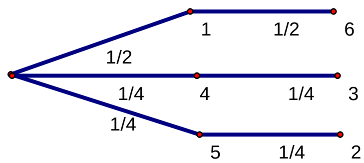
c. Find $A \cap B$ as set. Find $P(A \cap B)$.
$A \cap B$={55,66} so $P(A \cap B)=2/36=1/18$

d. Are A and B independent events? **No, $P(A)*P(B)=1/9*1/6=1/54 \neq 1/18= P(A \cap B)$**
Events A & B are dependent. P(A|B) = (1/18)/(1/6) = 1/3 > P(B) = 1/6 Same will be true for reverse situation, so the events positively reinforce each other.

4. In a game, each spinner is spun once and the results are added. Suppose that the outcome is 7. What is the chance that the first spinner was 1?



Use a partial tree to find the outcomes that we need and their probabilities.



outcome	prob
1+6	1/4
4+3	1/16
5+2	1/16

Use the partial tree to find P(first spinner is 1 | sum is 7):

$$= \frac{\mathbf{P(first\ spinner\ is\ 1\ and\ sum\ is\ 7)}}{\mathbf{P(\ sum\ is\ 7)}} = \frac{\mathbf{1/4}}{\mathbf{\frac{1}{4}+\frac{1}{16}+\frac{1}{16}}} = \frac{\mathbf{1/4}}{\mathbf{3/8}} = \frac{\mathbf{2}}{\mathbf{3}}$$

5. Four cards are pulled from a deck of 52 cards. Find the probability of obtaining

a. at least one club: **find complement (no club)** $1-\frac{\binom{39}{4}}{\binom{52}{4}}$	**b.** a pair**: choose face value of pair, then suits of pair cards, then the nonpair cards** $\frac{13\binom{4}{2}\binom{48}{2}}{\binom{52}{4}}$
c. 3 of a kind: **the same reasoning as for pair** $\frac{13\binom{4}{3}\binom{48}{1}}{\binom{52}{4}}$	d. 2 pair: **choose the 2 denominations, respective suits then nonpair card** $\frac{\binom{13}{2}\binom{4}{2}^2\binom{44}{1}}{\binom{52}{4}}$.
e. straight (4 cards whose face values are in order, an ace can be lower than 2 or higher than king): **choose starting face (all except Q&K) & suits for each card** $\frac{11*4^4}{\binom{52}{4}}\approx 1.04\%$	**f.** a flush (all 4 of the same suit): **choose suit and then face values for the 4 cards** $\frac{4*\binom{13}{4}}{\binom{52}{4}}\approx 1.06\%$
g. straight flush (both straight and flush): **choose common suit & starting face** $\frac{4*11}{\binom{52}{4}}\approx 0.016\%$	h. use your answers to e, f and g to determine whether getting a straight and getting a flush are independent events. If not, are they positively or negatively reinforcing? **P(flush|straight) =ans to g/ans to e≈*1.56%*** **Positively: The chance of flush increases by ~50%.**

6. A jar contains 3 chocolate chip cookies and x oatmeal cookies. Two cookies are pulled one at a time from the jar without replacement.
   a. Find an expression that represents the probability one cookie is chocolate chip and the next cookie is oatmeal.

$$\frac{3}{3+x}*\frac{x}{2+x}=\frac{3x}{(3+x)(2+x)}$$

b. Find an expression that represents the probability one cookie is chocolate chip and the other cookie is oatmeal, regardless of the order in which they come out.

$$\frac{3}{3+x}*\frac{x}{2+x}+\frac{x}{3+x}*\frac{3}{2+x}=\frac{6x}{(3+x)(2+x)} \text{ or } \frac{\binom{3}{1}\binom{x}{1}}{\binom{x+3}{2}}$$

c. If the chance of getting the event described in a. is 2/7, find an equation and solve to determine x.

$$\frac{3x}{(3+x)(2+x)}=\frac{2}{7} \textbf{ or } 21x=2(3+x)(2+x)=12+10x+2x^2 \textbf{ or } 0=2x^2-11x+12=(2x-3)*(x-4) \textbf{ or}$$

{3/2, 4} but answer must be an integer so x=4.

7. A 5 digit PIN number can begin with any digit (except zero) and the remaining digits have no restriction, but otherwise is selected randomly.
   a. Find the probability that the PIN code has no repeated digits, begins with a 7 and ends with an 8. **Outer digits are determined & middle 3 digits can't do repeats so 8 nPr 3/(9*10^4)=1/90**
   b. Find the probability of the PIN code is odd. **Last digit is 1, 3, 5, 7 or 9. P(O)=9*10^3*5/(9*10^4)=1/2**
   c. Find the conditional probability that the PIN code is odd given that the code has no repeated digits: **Make selections for the last and first digits which will have 5 and 8 possible choices respectively, then permutation for the middle 3 digits. Denominator is enumerated by counting choices for 1st digit then it is a permutation for remaining digits: P(O|NR)=P(O∩NR)/P(NR)=5*8*8 nPr 3/(9*9 nPr 4)=5*8/9^2**
   d. Are the events PIN is odd and PIN has no repeated digits independent? If not, are they positively or negatively reinforcing?
   No, P(O)=.5≠ P(O|NR)=40/81. Not allowing repeats has a slightly negative effect on the chance of being odd.

8. There are 12 top female runners in a marathon, 7 from Kenya and 5 from outside of Kenya. If they each have an equal chance getting any of the top 12 positions, find the chance that
   a. exactly 3 of the top 5 runners will be from Kenya

   $$P(X=3)=\frac{\binom{7}{3}\binom{5}{2}}{\binom{12}{5}}=\frac{7nCr3*5nCr2}{12nCr5}\approx 0.44$$

   b. all 5 of the top runners will be from Kenya

   $$P(X=5)=\frac{\binom{7}{5}\binom{5}{0}}{\binom{12}{5}}=\frac{7*6*5*4*3}{12*11*10*9*8}=\frac{7}{11*3*8}=\frac{7}{264}\approx 0.027$$

   c. at least 3 of the top 5 runners will be from Kenya.

   $$P(X\geq 3)=\frac{\binom{7}{3}\binom{5}{2}}{\binom{12}{5}}+\frac{\binom{7}{4}\binom{5}{1}}{\binom{12}{5}}+\frac{\binom{7}{5}\binom{5}{0}}{\binom{12}{5}}\approx 0.69$$