MAT2440 Practice Final Exam Halleck Spring 2019 Name: ____________________

- Book and notes are prohibited except for a single sheet (back and front) with hand-written formulae/notes. Submit formula sheet with your exam for up to 5 extra pts.
- You may write on test page. However, put all your work and answers into the blue book.
- No credit will be given for any answer that is not backed up with work.
- The use of any electronic devices except a graphing calculator is strictly prohibited.

1. Let C(x, y) be the statement "x finds y charming," where the domain for x and y consists of all people in the world. Use quantifiers to express each of the following statements.
   (a) Everyone finds themselves charming.
   (b) Someone finds Jerry charming.
   (c) There is a person that finds everyone charming.
   (d) If you find Max charming, then you will also find Gina charming.

(a) $\forall x \; C(x,x)$

(b) $\exists x \; C(x, \text{Jerry})$

(c) $\exists x \forall y \; C(x,y)$

(Note: $\forall y \exists x \; C(x,y)$ is incorrect!!)

(d) $\forall x \; C(x, \text{Max}) \rightarrow C(x, \text{Gina})$

2. Answer each of the following questions.
   (a) Show that $p \rightarrow q$ is logically equivalent to $\neg q \rightarrow \neg p$ using a truth table.

p	q	$p \rightarrow q$ *	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$ *
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
1	1	1	0	0	1

since the 2 cols marked * are identical, the respective statements are $\cong$

   (b) Show that $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$ without using a truth table.

$\neg p \rightarrow (q \rightarrow r) \equiv p \vee (\neg q \vee r)$ and $q \rightarrow (p \vee r) \equiv \neg q \vee (p \vee r)$

Using commutative and associative laws for $\vee$, these expressions are equivalent (remove parentheses and rearrange terms).

   (c) Show that $\neg (p \rightarrow q) \rightarrow \neg q$ is a tautology without using a truth table.

$\neg (p \rightarrow q) \rightarrow \neg q \equiv (p \rightarrow q) \vee \neg q \equiv (\neg p \vee q) \vee \neg q \equiv \neg p \vee (q \vee \neg q) \equiv \neg p \vee T \equiv T$

3. Prove that for all integers n, n is even if and only if 5n + 3 is odd.

(⇒) n even means n=2k, k ∈Z; 5n + 3 = 5(2k) + 3 = 10k + 2 + 1 = 2(5k + 1) + 1 so 5n + 3 is odd

(⇐) By contradiction: we assume that 5n +3 is odd but then we assume the opposite of the conclusion, namely that n is odd, i.e., n =2k+1. But then 5n + 3 = 5(2k + 1) + 3 = 10k + 8 = 2 (5k + 4) so 5n + 3 is even, which contradicts the assumption so in fact the conclusion must be true, namely n is even.

4. Translate the following specifications into English where:

F(p): "printer p is out of service" B(p): "printer p is busy" Q(j): "print job j is queued"

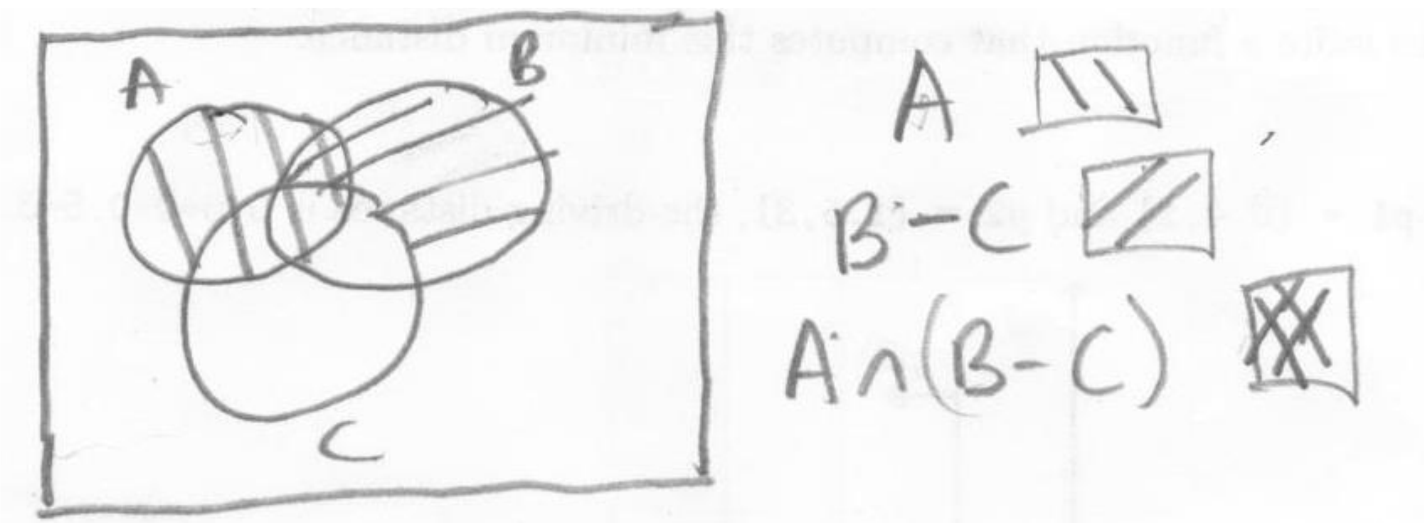
(a) $\exists j\, Q(j) \rightarrow \exists p\, (F(p) \vee B(p))$ If there is job queued, then some printer must be busy or out of service.

(b) $\forall j\, Q(j) \rightarrow \forall p\, F(p)$ If all jobs are queued then all printers are out of service.

NOTE: assume that a print job that is being processed is no longer queued.

5. (a) Determine cardinality of the set A= $\{\emptyset, \{a\}, \{\emptyset, a\}\}$ as well as each member of set A.
|A|=3, $|\emptyset|$=0, $|\{a\}|$=1, $|\{\emptyset, a\}|$=2

(b) Draw the Venn diagram for the following combination of the sets A, B, and C.
$A \cap (B - C)$ (shade A and $B - C$ as intermediate steps and use a legend)



(c) Is it true that $A \cap (B - C) = (A \cap B) - (A \cap C)$? Use a ~~truth~~ membership table.

A	B	C	B–C	$* A \cap (B - C)$	A∩B	A∩C	$*(A \cap B) - (A \cap C)$
1	1	1	0	0	1	1	0
1	1	0	1	1	1	0	1
1	0	1	0	0	0	1	0
1	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0

The two columns marked * are identifical. ∴the identity being investigated is true.

6. Determine if the following functions are 1-1 and/or onto:

(a) $f: Z \rightarrow Z, f(x) = 3x^3 - 2$

f is 1-1: Suppose f(a) = f(b), then $3a^3 - 2 = 3b^3 - 2$ and then $a^3 = b^3$. We now apply the cube root to both sides. Cube root is a 1-1 function (its graph has a horizontal line only pass through 1 point), so this is legitimate. Hence we get a=b.

f is NOT onto: If we select almost any integer as an image, the preimage is not an integer. For instance, $3x^3 - 2 = 0$ means $x = \sqrt[3]{\frac{2}{3}}$ which is not an integer. The range is just a very limited set of integers:

{…., f(-1)=-5, f(0)=-2,f(1)= 1, f(2)=22,…} [For a function to be onto, the range must coincide with the codomain.]

(b) $g: R \to Z, g(x) = \lfloor x/2 \rfloor + 6$

g is NOT 1-1: A horizontal line at any integer passes through more than one point. g(0)=g(.1)= 6.

g is onto: The set of even integers maps to all the integers: Given n∈Z, let m=2(n-6), then g(m)=n.

7. Use insertion sort with input 2, 1, 4, 3, 5, showing as separate steps the comparisons, rotations and insertions. You should have approximately 5+4+3+2+1 steps, each showing all or a portion of the list.

working	Insert #	reserve	comments
2_	1	435	1<2 T
_2	1	435	rotate
12		435	insert
12_	4	35	4<1 F
12_	4		4<2 F
124		35	a) insert
124_	3	5	b) 3<1 F
124_	3	5	c) 3<2 F
124_	3	5	d) 3<4 T
12_4	3	5	e) rotate
1234		5	f) insert
1234_	5		g) 5<1 F
1234_	5		h) 5<2 F
1234_	5		i) 5<3 F
1234_	5		j) 5<4 F
12345			k) insert

8. The ISBN-10 of *Mathematical Modeling and Computer Simulation* is 0-534-Q8478-1, where *Q* is a digit. Find the value of *Q*. Use the Euclidean Algorthm (EA) to find the appropriate inverse.
1*0+2*5+3*3+ 4*4+5*Q+6*8+7*4+8*7+9*8+10*1=249+5*Q≡0 mod 11 so 5*Q≡-249 mod 11
-249=-23*11+4 so -249 ≡4 mod 11 and 11=2*5+1 or 11-2*5=1 so inv(5) ≡ -2 ≡ 9 mod 11. Hence, Q≡ 9*4 ≡ 36 ≡ 3 mod 11. As a check, 249+ 5*3= 264 which is indeed 0 mod 11 (2-6+4=0)

9. If encryption function is *f* (*p*) = (7*p* + 13) **mod** 26, decrypt TZURCQKINZ: translate letters into #s, apply appropriate decryption function (use EA to find inverse of 7) & then translate #s back into letters.
26=3*7+5, 7=1*5+2 and 5=2*2+1.
26-3***7**=**5**, **7**-1***5**=**2** and **5**-2***2**=1.
5-2*(**7**-1***5**)=1 or **5**-2***7**+2***5**=1 or -2***7**+3***5**=1
-2***7**+3(**26**-3***7**)=1 or -2***7**+3***26**-9***7**=1 or 3***26**-11***7**=1
Hence inv(7) ≡ -11≡ 15 mod 26 As a check 7*15=105≡ 1 mod 26
The decryption function is p=15(c-13)mod26 where c is the the number associated with the CT.

CT	**NUM**	**DEC**	**PT**
T	19	12	M
Z	25	24	Y
U	20	1	B
R	17	8	I
C	2	17	R
Q	16	19	T
K	10	7	H
I	8	3	D
N	13	0	A
Z	25	24	Y

By the way, the cyphertext's last 2 letters have been changed. As Daniel had demonstrated in class, they were in error on the practice exam sheet.

10. Choose ONE of the following 2 INDUCTION proof problems:
(i)(a) Find formula for $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n(n+1)}$ by examining values of this expression for small values of n
(b) Prove the formula you conjectured in part (a).

(a) The first 3 sums are ½ , 2/3, ¾ (when simplified) so we conjecture the formula s(n)= n/(n+1)

(b) Basis step:1/(1*2)= ½ = 1/(1+1)

Induction step: assume true for n=k, show true for n=k+1

s(k+1)=s(k)+1/((k+1)(k+2))= k/(k+1)+ 1/((k+1)(k+2)) =(k(k+2)+1)/ ((k+1)(k+2)) = (k^2+2k+1)/ ((k+1)(k+2)) = (k+1)^2/((k+1)(k+2)) = (k+1)/ (k+2) which completes the induction step

We conclude that the conjectured formula s(n)= n/(n+1) holds for all n>0

(ii) Prove that $3^n < n!$ if *n* is greater than 6.
Basis step: 3^7=2187<5040=7!
Induction step: assume true for n=k, show true for n=k+1
3^(k+1)=3*3^k<3(k!)<(k+1)k!=(k+1)! (6<k, therefore 3<6<k<k+1)
Which completes the induction step
We conclude that the inequality holds for all n>6